MATHEMATICS - II A
SECOND YEAR

CONTENTS

MATHEMATICS - II A

EXPECTED WIGHTAGE OF MARKS CHAPTERWISE

S.NO	CHAPTER	VSAQ (2M)	SAQ(4M)	LAQ (7M)	TOTAL
1	COMPLEX NUMBERS	2(2)	4(1)		8
2	DE MOIVRE'S THEOREM	2(1)		7(1)	9
3	QUADRATIC EXPRESSIONS	2(1)	4(1)		6
4	THEORY OF EQUATIONS	2(1)		7(1)	9
5	PERMUTATIONS AND COMBINATIONS	2(2)	4(2)		12
6	BINOMIAL THEOREM	2(1)		7(2)	16
7	PARTIAL FRACTIONS		4(1)		4
8	MEASURES OF DISPERSION	2(1)		7(1)	9
9	PROBABILITY		4(2)	7(1)	15
10	RANDOM VARIABLES	2(1)		7(1)	9
		20(10)	20(7)	35(7)	97

Chapter - 1

COMPLEX NUMBERS

Weightage : (2 + 2 + 4)

Key Concepts:

$\rightarrow$A complex number is an ordered pair of real numbers. It is denoted by (a, b); $a \in R, b \in R$,

z = a+ib, Re(z)=a and Im(z)

$\rightarrow$Two complex numbers z_1=a+ib, z_2=c+id are said to be equal if a=c, b=d

$\rightarrow$Algebra of complex numbers z_1=a+ib, z_2=c+id then

(a) $z=z_1+z_2=(a+c)+i(b+d)$

(b) $z=z_1-z_2=(a-c)+i(b-d)$

(c) $z=z_1.z_2=(ac-bd)+i(ad+bc)$

(d) $z=\frac{z_1}{z_2}=\frac{ac+bd}{c^2+d^2}+i\frac{(bc-ad)}{c^2+d^2}$

$\rightarrow$If z=a+ib then conjugate of complex number $\bar{z}=a-ib$

$\rightarrow$If z=a+ib then additive inverse of a complex number –z = - a - ib

$\rightarrow$If z=a+ib then $|z|=\sqrt{a^2+b^2}$

$\rightarrow$If z=a+ib then $\sqrt{z}=\sqrt{a+ib}=\pm\left(\sqrt{\frac{\sqrt{a^2+b^2}+a}{2}}+i\sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}\right)$ if b>0

$\rightarrow$If z=a+ib then modulus-amplitude or polar form of a+ib=r$(\cos\theta+i\sin\theta)$

Where $r=\sqrt{a^2+b^2}, \cos\theta=\frac{a}{r}, \sin\theta=\frac{b}{r}$ where $\theta \in (-\pi,\pi]$

$\rightarrow \cos\theta+i\sin\theta$ is simply denoted by '$\text{cis}\theta$'

$\rightarrow$Z=a+ib then Argz= $\tan^{-1}\frac{im(z)}{Re(z)}=\tan^{-1}\frac{b}{a}$

$\rightarrow$Arg($z_1.z_2$)=Argz_1 +Argz_2

$\rightarrow$Arg(z1/z2)=Argz_1 -Argz_2

$\rightarrow i^2 = -1, i^3 = -i, i^4 = 1, i^5 = i$

$\rightarrow i=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}=\text{cis}\frac{\pi}{2}$

$\rightarrow -i=\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)=\text{cis}\left(-\frac{\pi}{2}\right)=$

$\rightarrow$1=cos0+isin0=cis(0)

$\rightarrow -1=\cos\pi+i\sin\pi=\text{cis}(\pi)=$

Level-1

Very Short Answer Questions:

1. **Find the additive inverse of (-6,5)+(10,-4)**

Sol. -6+5i+10-4i=4+i

$\therefore$ Additive inverse of 4+i is -4-i

$\because$ Additive inverse of a+ib=-a-ib

2. **Find the multiplicative inverse of 7+24i**

Sol. Multiplicative inverse of 7+24i is $\frac{7-24i}{625}$

$\because$ Multiplicative inverse of a+ib= $\frac{a-ib}{a^2+b^2}$

3. **Find the complex conjugate of (3+4i)(2-3i)**

Sol. $(3+4i)(2-3i)=6-9i+8i-12i^2=18-i$ $\quad\because i^2=-1$

$\therefore$ Complex conjugate of 18-i is 18+i

$\because$ Complex conjugate of a+ib is a-ib

4. **If** $z=(\cos\theta,\sin\theta)$ **find** $z-\frac{1}{z}$

Sol. $z-\frac{1}{z}=\cos\theta+i\sin\theta-(\cos\theta-i\sin\theta)$

$=\cos\theta+i\sin\theta-\cos\theta+i\sin\theta$

$=2i\sin\theta$ $\quad\because z=\cos\theta+i\sin\theta \Rightarrow \frac{1}{z}=\cos\theta-i\sin\theta$

5. **Find the real and imaginary parts of the complex number** $\frac{a+ib}{a-ib}$

Sol. $\frac{a+ib}{a-ib}=\frac{(a+ib)^2}{(a-ib)(a+ib)}=\frac{a^2+i^2b^2+2iab}{a^2-i^2b^2}=\frac{a^2-b^2+2iab}{a^2+b^2}=\frac{a^2-b^2}{a^2+b^2}+\frac{2iab}{a^2+b^2}$ $\quad\boxed{\therefore i^2=-1}$

$\therefore$ The real part is $\frac{a^2-b^2}{a^2+b^2}$ and imaginary part is $\frac{2ab}{a^2+b^2}$

6. Find the square root of 7+24i

Sol. $\because \sqrt{a+ib}=\pm\left[\frac{\sqrt{a^2+b^2}+a}{2}+i\sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}\right]$ if b>0

$$\sqrt{7+24i}=\pm\left[\sqrt{\frac{\sqrt{7^2+24^2}+7}{2}}+i\sqrt{\frac{\sqrt{7^2+24^2}-7}{2}}\right]$$

$$=\left[\sqrt{\frac{\sqrt{49+576}+7}{2}}+i\sqrt{\frac{\sqrt{49+576}-7}{2}}\right]$$

$$=\pm\left[\sqrt{\frac{\sqrt{625}+7}{2}}+i\sqrt{\frac{\sqrt{625}-7}{2}}\right]$$

$$=\pm\left[\sqrt{\frac{25+7}{2}}+i\sqrt{\frac{25-7}{2}}\right]$$

$$=\pm\left[\sqrt{16}+i\sqrt{9}\right]$$

$=\pm[4+3i]$

7. **If z=2-3i show that z^2-4z+13=0**

Sol. Given that z=2-3i

z-2=-3i

squaring on both sides

$(z-2)^2=(-3i)^2$

$Z^2-4z+4=9i^2$

$Z^2-4z+4=-9$

$Z^2-4z+13=0$

$\because (a-b)^2=a^2-2ab+b^2$

$\because i^2=-1$

8. **If** $z\neq 0$**. Find** $\text{Arg}z+\text{Arg}\bar{z}$

Sol. Z=a+ib, $\text{Arg}z=\theta$ and $\text{Arg}\bar{z}=-\theta$

$\text{Arg}z+\text{arg}\bar{z}=\theta-\theta=0$

$\because$ I)$\text{Arg}z=\theta\Rightarrow\text{Arg}\bar{z}=-\theta$

9. **Find the polar or modulus-amplitude form of following complex numbers**

(i) $1+i\sqrt{3}$ **(ii) -1-i**

Sol. (i) Let $a+ib=1+i\sqrt{3}$

Here a = 1, b= $\sqrt{3}$

Now $r=\sqrt{a^2+b^2}=\sqrt{1^2+\left(\sqrt{3}\right)^2}=\sqrt{1+3}=\sqrt{4}=2$

$$\cos\theta=\frac{a}{r}=\frac{1}{2},\sin\theta=\frac{b}{r}=\frac{\sqrt{3}}{2}$$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}\Rightarrow\theta=\tan^{-1}\sqrt{3}$$

$$\theta=\frac{\pi}{3}$$

'θ' lies in I quadrant and $\theta=\frac{\pi}{3}\in[-\pi,\pi]$

$\therefore$ Mod-amplitude form of $1+i\sqrt{3}=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

$\because$ Polar or mod-Amplitude form of $a+ib=r(\cos\theta+i\sin\theta)$

(ii) -1-i

Let a+ib=-1-i

Here a=-1, b=-1

Now $r=\sqrt{a^2+b^2}=\sqrt{(-1)^2+(-1)^2}=\sqrt{2}$

$$\cos\theta=\frac{a}{r}=\frac{-1}{2},\sin\theta=\frac{b}{r}=\frac{-1}{2}$$

'θ' lies in III quadrant and $\theta=\frac{\pi}{4}-\pi$

$$\theta=\frac{-3\pi}{4}$$

$\therefore$ Mod-amplitude form of $-1-i=\sqrt{2}\left(\cos\left(\frac{-3\pi}{4}\right)+i\sin\left(\frac{-3\pi}{4}\right)\right)$

10. **If z_1=-1 and z_2=i then find** $\mathrm{Arg}\frac{z_1}{z_2}$

Sol. $z_1=-1=\cos\pi+i\sin\pi \Rightarrow \mathrm{Arg}z_1=\pi$

$z_2=i=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2} \Rightarrow \mathrm{Arg}z_2=\frac{\pi}{2}$

$\mathrm{Arg}\frac{z_1}{z_2}=\mathrm{Arg}z_1=\mathrm{Arg}z_2$

$=\pi-\frac{\pi}{2}=\frac{\pi}{2}$

$\because \mathrm{Arg}\frac{z_1}{z_2}=\mathrm{Arg}z_1-\mathrm{Arg}z_2$

11. **I) z_1=-1 and z_2=-I, then find Arg(z_1z_2)**

Sol. $z_1=-1=\cos\pi+i\sin\pi \Rightarrow \mathrm{Arg}z_1=\pi$

$z_2=-i=\cos\left(\frac{-\pi}{2}\right)+i\sin\left(\frac{-\pi}{2}\right) \Rightarrow \mathrm{Arg}z_2=\frac{-\pi}{2}$

$\mathrm{Arg}(z_1z_2)=\mathrm{Arg}z_1+\mathrm{Arg}z_2$ $\quad\because \mathrm{Arg}z_1z_2=\mathrm{Arg}z_1+\mathrm{Arg}z_2$

$=\pi-\frac{\pi}{2}=\frac{\pi}{2}$

12. **I) z=x+iy and |z|=1, then find the locus of z**

Sol. Given z=x+iy and |z|=1

$\Rightarrow |x+iy|=1$

$\Rightarrow \sqrt{x^2+y^2}=1 \Rightarrow x^2+y^2=1$ $\quad\because |x+iy|=\sqrt{x^2+y^2}$

13. **If $\mathrm{Arg}\overline{z_1}$ and $\mathrm{Arg}z_2$ are $\frac{\pi}{5}$ and $\frac{\pi}{3}$ respectively, then find ($\mathrm{Arg}z_1+\mathrm{Arg}z_2$)**

Sol. Given that $\mathrm{Arg}\overline{z_1}=\frac{\pi}{5} \Rightarrow \mathrm{Arg}z_1=\frac{-\pi}{5}$

$\mathrm{Arg}z_2=\frac{\pi}{3}$ $\quad\because \mathrm{Arg}z=\theta \Rightarrow \mathrm{Arg}\bar{z}=-\theta$

$\therefore \mathrm{Arg}z_1+\mathrm{Arg}z_2=\frac{-\pi}{5}+\frac{\pi}{3}=\frac{2\pi}{15}$

14. **Find the least positive integer n, satisfying** $\left(\frac{1+i}{1-i}\right)^n=1$

Sol. Given that $\left(\frac{1+i}{1-i}\right)^n=1 \Rightarrow \left(\frac{1+i}{1-i}\times\frac{1+i}{1+i}\right)^n=1$

$\Rightarrow \left[\frac{(1+i)^2}{1^2-i^2}\right]^n=1 \Rightarrow \left(\frac{1+i^2+2i}{1+1}\right)^n=1$

$\Rightarrow \left(\frac{1-1+2i}{2}\right)^n=1 \Rightarrow \left(\frac{2i}{2}\right)^n=1$

$\Rightarrow i^n=1 \Rightarrow i^n=i^4$

$\therefore$ n=4 which is least

15. **If |z+ai|=|z-ai| find the locus of z**

Sol. Let z=x+iy

Given |z+ai|=|z-ai|

$\Rightarrow |x+iy+ai|=|x+iy-ai|$

$\Rightarrow |x+i(y+a)|=|x+i(y-a)|$

$\Rightarrow \sqrt{x^2+(y+a)^2}=\sqrt{x^2+(y-a)^2}$ $\qquad \because |x+iy|=\sqrt{x^2+y^2}$

SOBS

$\Rightarrow x^2+(y+a)^2=x^2(y-a)^2$

$\Rightarrow x^2+y^2+a^2+2ay=x^2+y^2+a^2-2ay$

$\Rightarrow 4ay=0 \Rightarrow y=0$

$\therefore$ Locus of z is x-axis

16. **If** $(x-iy)^{\frac{1}{3}}=a-ib$ **, then prove that** $\frac{x}{a}+\frac{y}{b}=4\left(a^2-b^2\right)$

Sol. Given $(x-iy)^{\frac{1}{3}}=a-ib$

$(x-iy)=(a-ib)^3$

$x-iy=a^3-3a^2ib+3ai^2b^2-i^3b^3$ $\qquad \because (a-b)^3=a^3-3a^2b+3ab^2-b^3$

$x-iy=a^3-3ab^2-3a^2ib+ib^3$

$x-iy=a^3-3ab^2-i\left(3a^2b-b^3\right)$ $\qquad \because i^2=-1$

Equations real & imaginary parts

$x=a^3-3ab^2;\ y=3a^2b-b^3$

$= a\left(a^2-3b^2\right);\ =b\left(3a^2-b^2\right)$

$\frac{x}{a}=a^2-3b^2 \qquad \frac{y}{b}=3a^2-b^2$

Then $\frac{x}{a}+\frac{y}{b}=a^2-3b^2+3a^2-b^2$

$= 4a^2-4b^2$

$=4\left(a^2-b^2\right)$

17. **If** $(a+ib)^2=x+iy$ **find x^2+y^2**

Sol. Given $(a+ib)^2=x+iy$

$a^2+i^2b^2+2abi=x+iy$

$a^2-b^2+2abi=x+iy$

equating real and imaginary parts

$x=a^2-b^2,\ y=2ab$

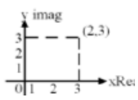
Then $x^2+y^2=(a^2-b^2)+(2ab)^2$

$=(a^2-b^2)+4a^2b^2$

$x^2+y^2=(a^2+b^2)^2$ $\qquad \because (a-b)^2+4ab=(a+b)^2$

18. **Represent the complex number 2+3i in Argand plane**

Sol. The complex number 2+3i in the Argand plane is represented as



***Level-2*:**

1. **Write the complex conjugate of** $\frac{5i}{7+i}$

Sol. $\frac{5i}{7+i}=\frac{5i(7-i)}{(7+i)(7-i)}=\frac{5\left(7i-i^2\right)}{49-i^2}=\frac{5(7i+1)}{49+1}=\frac{5(7i+1)}{50}=\frac{7i+1}{10}$

$\therefore$ complex conjugate of $\frac{1+7i}{10}=\frac{1-7i}{10}$

2. **Simplify $i^2+i^4+i^6+......+(2n+1)$ terms**

Sol. $i^2+i^4+i^6+i^8+.........+(2n+1)$ terms

$=i^2+(i^2)^2+(i^2)^3+(i^2)^4+........+$ odd no.of terms

$=-1+1-1+1........-1=-1$

3. **Find the square root of** $-47+i8\sqrt{3}$

Sol. $\because \sqrt{a+ib}=\pm\left(\sqrt{\frac{\sqrt{a^2+b^2}+a}{2}}+i\sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}\right)$ if b>0

$$\sqrt{-47+i8\sqrt{3}}=\pm\left(\sqrt{\frac{\sqrt{(-47)^2+\left(8\sqrt{3}\right)^2}-47}{2}}+i\sqrt{\frac{\sqrt{(-47)^2+\left(8\sqrt{3}\right)^2}+47}{2}}\right)$$

$$=\pm\left(\sqrt{\frac{\sqrt{2209+192}-47}{2}}+i\sqrt{\frac{\sqrt{2209+192}+47}{2}}\right)$$

$$=\pm\left(\sqrt{\frac{\sqrt{2401}-47}{2}}+i\sqrt{\frac{\sqrt{2401}+47}{2}}\right)$$

$$=\pm\left(\sqrt{\frac{49-47}{2}}+i\sqrt{\frac{49+47}{2}}\right)$$

$$=\pm\left(\sqrt{\frac{2}{2}}+i\sqrt{\frac{96}{2}}\right)$$

$$=\pm\left(\sqrt{1}+i\sqrt{48}\right)$$

$$==\pm\left(1+i4\sqrt{3}\right)$$

4. **Find the polar form of following complex numbers**

(i) $-1-i\sqrt{3}$ **(ii)** $-\sqrt{7}+i\sqrt{21}$

Sol. (i) $-1-i\sqrt{3}$

Let $x+iy=-1-i\sqrt{3}$

Here $x=-1, y=-\sqrt{3}$

Now $r=\sqrt{x^2+y^2}=\sqrt{(-1)^2+\left(-\sqrt{3}\right)^2}=\sqrt{1+3}=\sqrt{4}=2$

$=\cos\theta=\frac{x}{r}\Rightarrow\cos\theta=\frac{-1}{2}$

$=\sin\theta=\frac{y}{r}\Rightarrow\sin\theta=\frac{-\sqrt{3}}{2}$

θ lies in III quadrant and $\theta=\frac{\pi}{3}-\pi$

$=\frac{-2\pi}{3}$

$\therefore$ Polar form of $-1-i\sqrt{3}=r(\cos\theta+i\sin\theta)$

$$=2\left(\cos\left(\frac{-2\pi}{3}\right)+i\sin\left(\frac{-2\pi}{3}\right)\right)$$

(ii) $z=-\sqrt{7}+i\sqrt{21}$

Sol. Let $x+iy=-\sqrt{7}+i\sqrt{21}$

Here $x=-\sqrt{7}, y=\sqrt{21}$

Now $r=\sqrt{x^2+y^2}=\sqrt{\left(-\sqrt{7}\right)^2+\left(\sqrt{21}\right)^2}=\sqrt{7+21}=\sqrt{28}=2\sqrt{7}$

Hence $\cos\theta=\frac{x}{r}\Rightarrow\cos\theta=\frac{-\sqrt{7}}{2\sqrt{7}}=-\frac{1}{2}$

$\sin\theta=\frac{y}{r}\Rightarrow\sin\theta=\frac{\sqrt{21}}{2\sqrt{7}}=\frac{\sqrt{3}\times\sqrt{7}}{2\sqrt{7}}=\frac{\sqrt{3}}{2}$

$\therefore\theta$ lies in II quadrant and $\theta=\pi-\frac{\pi}{3}$

$\therefore$ Polar form of $-\sqrt{7}+i\sqrt{21}=r\left(\cos\theta+i\sin\theta\right)=2\sqrt{7}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

5. **If z = 3-5i, the show that $z^3-10z^2+58z-136=0$**

Sol. Given that z = 3-5i

3-z=5i SOBS

$(3-z)^2=(5i)^2$

$9+z^2-6z=25i^2\Rightarrow 9+z^2-6z=-25$

$\Rightarrow z^2-6z+9+25=0$

$\Rightarrow z^2-6z+34=0$

Now $z^3-10z^2+58z-136=z(z^2-6z+34)-4(z^2-6z+34)$

$=z(0)+4(0)=0$

6. **If the amplitude of (z-1) is $\frac{\pi}{2}$ then find locus of z.**

Sol. Let z=x+iy then z-1=x+iy-1=(x-1)+iy

Amp(z-1)= $\frac{\pi}{2}$

$\tan^{-1}\left(\frac{y}{x-1}\right)=\frac{\pi}{2}$

$\Rightarrow\frac{y}{x-1}=\tan\frac{\pi}{2}\Rightarrow\frac{y}{x-1}=\infty\Rightarrow\frac{y}{x-1}=\frac{1}{0}\Rightarrow x-1=0$

$\therefore$ Locus of z is x-1=0

7. **If (1-i)(2-i)(3-i)…….(1-ni)=x-iy then prove that 2.5.10……($1+n^2$)=x^2+y^2**

Sol. (1-i)(2-i)(3-i)…….(1-ni)=x-iy

Applying mod on both sides

$\Rightarrow$ |(1-i)||(2-i)||(3-i)|……….|(1-ni)|=|x-iy| $\quad\because |x+iy|=\sqrt{x^2+y^2}$

$\Rightarrow\sqrt{1^2+(-1)^2}\sqrt{2^2+(-1)^2}\sqrt{3^2+(-1)^2}\ldots\ldots\ldots\sqrt{1^2+(-n)^2}=\sqrt{x^2+y^2}$

$\Rightarrow\sqrt{2}\sqrt{5}\sqrt{10}\ldots\ldots\sqrt{1+n^2}=\sqrt{x^2+y^2}$

$\Rightarrow 2.5.10\ldots\ldots\left(1+n^2\right)=x^2+y^2$

8. If $\left(\sqrt{3}+i\right)^{100}=2^{99}(a+ib)$ **then show that $a^2+b^2=4$**

Sol. $\left|\left(\sqrt{3}+i\right)\right|^{100}=2^{99}|a+ib|$

$\left[\sqrt{\left(\sqrt{3}\right)^2+1^2}\right]^{100}=2^{99}\sqrt{a^2+b^2}$

$\left(\sqrt{3+1}\right)^{100}=2^{99}\sqrt{a^2+b^2}$

$\left(\sqrt{4}\right)^{100}=2^{99}\sqrt{a^2+b^2}$

$2^{100}=2^{99}\sqrt{a^2+b^2}$

$\frac{2^{100}}{2^{99}}=\sqrt{a^2+b^2}$

$2=\sqrt{a^2+b^2}$ SOBS

$\therefore$ $a^2+b^2=4$

Short Answer Questions (4 Marks)

Level-1 **:**

1. **If** $x+iy=\frac{1}{1+\cos\theta+i\sin\theta}$ **then show that $4x^2-1=0$**

Sol. Given that $x+iy=\frac{1}{1+\cos\theta+i\sin\theta}$

$=\frac{1}{1+\cos\theta+i\sin\theta}\times\frac{1+\cos\theta-i\sin\theta}{1+\cos\theta-i\sin\theta}$

$=\frac{1+\cos\theta-i\sin\theta}{(1+\cos\theta)^2-(i\sin\theta)^2}$ Rationalise the denominator

$=\frac{1+\cos\theta-i\sin\theta}{1+\cos^2\theta+2\cos\theta-i^2\sin^2\theta}$

$=\frac{1+\cos\theta-i\sin\theta}{1+\cos^2\theta+2\cos\theta+\sin^2\theta}$ $\because i^2=-1$

$=\frac{1+\cos\theta-i\sin\theta}{2+2\cos\theta}$

$=\frac{1+\cos\theta}{2(1+\cos\theta)}-i\frac{\sin\theta}{2(1+\cos\theta)}$

$=\frac{1}{2}-\frac{i\sin\theta}{2(1+\cos\theta)}$

Equating real part of x+iy

$=x=\frac{1}{2}\Rightarrow 2x=1\Rightarrow 4x^2=1\Rightarrow 4x^2-1=0$

2. **If** $x+iy=\frac{3}{2+\cos\theta+i\sin\theta}$ **then show that $x^2+y^2=4x-3$**

Sol. Given that $x+iy=\frac{3}{2+\cos\theta+i\sin\theta}$

$=\frac{3}{2\cos\theta+i\sin\theta}\times\frac{2\cos\theta-i\sin\theta}{2+\cos\theta-i\sin\theta}$ Rationalise the Denominator

$$=\frac{3(2+\cos\theta-i\sin\theta)}{(2+\cos\theta)^2-(i\sin\theta)^2}$$

$$=\frac{6+3\cos\theta-i(3\sin\theta)}{4+\cos^2\theta+4\cos\theta-i^2\sin^2\theta}$$

$$=\frac{(6+3\cos\theta)-i(3\sin\theta)}{4+\cos^2\theta+4\cos\theta+\sin^2\theta} \qquad \because i^2=-1$$

$$x+iy=\frac{6+3\cos\theta-i3\sin\theta}{5+4\cos\theta}$$

$$=\frac{6+3\cos\theta}{5+4\cos\theta}-i\frac{3\sin\theta}{5+4\cos\theta}$$

Equating real and imaginary parts

$$x=\frac{6+3\cos\theta}{5+4\cos\theta},y=\frac{-3\sin\theta}{5+4\cos\theta}$$

L.H.S. $=x^2+y^2=\left(\frac{6+3\cos\theta}{5+4\cos\theta}\right)^2+\left(\frac{-3\sin\theta}{5+4\cos\theta}\right)^2$

$$=\frac{36+9\cos^2\theta+36\cos\theta+9\sin^2\theta}{(5+4\cos\theta)^2}$$

$$=\frac{36+9(\cos^2\theta+\sin^2\theta)+36\cos\theta}{(5+4\cos\theta)^2}$$

$$=\frac{45+36\cos\theta}{(5+4\cos\theta)^2}$$

$$=\frac{9(5+4\cos\theta)}{(5+4\cos\theta)^2}=\frac{9}{5+4\cos\theta}$$

R.H.S.=4x-3= $4\left(\frac{6+3\cos\theta}{5+4\cos\theta}\right)-3$

$$=\frac{24+12\cos\theta-15-12\cos\theta}{5+4\cos\theta}$$

$$=\frac{9}{5+4\cos\theta}$$

$\therefore$ L.H.S.=R.H.S. i.e, $x^2+y^2=4x-3$

3. **If the real part of $\frac{z+1}{z+i}$ is 1, then find the locus of z**

Sol. Let z=x+iy then $\frac{z+1}{z+i}=\frac{x+iy+1}{x+iy+i}=\frac{(x+1)+iy}{x+i(y+1)}$

$$=\frac{[(x+1)+iy]}{x+i(y+1)}\times\frac{x-i(y+1)}{x-i(y+1)}$$

$$=\frac{x(x+1)+ixy-i(x+1)(y+1)-i^2y(y+1)}{x^2-i^2(y+1)^2}$$

$$=\frac{x(x+1)+y(y+1)-i[(x+1)(y+1)-xy]}{x^2+(y+1)^2} \qquad \because i^2=-1$$

$$=\frac{x(x+1)+y(y+1)}{x^2+(y+1)^2}-i\left[\frac{(x+1)(y+1)-xy}{x^2+(y+1)^2}\right]$$

$$x+iy=\frac{x(x+1)+y(y+1)}{x^2+(y+1)^2}-i\frac{[(x+1)(y+1)-xy]}{x^2+(y+1)^2}$$

equating real part = 1

$$\frac{x(x+1)+y(y+1)}{x^2+(y+1)^2}=1$$

$$x^2+x+y^2+y=x^2+y^2+1+2y$$

$$x-y-1=0$$

4. **If z = x+iy and if the point P in the Argand plane represents z. Find the locus of z satisfying the equation |z-3+i|=4**

So. Given z=x+iy

$|z-3+i|=4 \Rightarrow |x+iy-3+i|=4$

$|(x-3)+i(y+1)|=4$

$\sqrt{(x-3)^2+(y+1)^2}=4$ $\quad\quad \because |x+iy|=\sqrt{x^2+y^2}$

SOBS

$(x-3)^2+(y+1)^2=16$

$x^2+9-6x+y^2+1+2y=16$

$x^2+y^2-6x+2y-6=0$

$\therefore$ Locus of z represents a circle

5. **If the amplitude of $\frac{z-2}{z-6i}=\frac{\pi}{2}$ find the locus of z**

Sol. Let z=x+iy then

$$\frac{z-2}{z-6i}=\frac{x+iy-2}{x+iy-6i}=\frac{x-2+iy}{x+i(y-6)}=\frac{x-2+iy}{x+i(y-6)}\times\frac{x-i(y-6)}{x-i(y-6)}$$

$$\frac{[(x-2)+iy][x-i(y-6)]}{x^2-i^2(y-6)^2}=\frac{(x-2)x-i(x-2)(y-6)+ixy-i^2y(y-6)}{x^2+(y-6)^2}$$

$$\frac{(x-2)x+y(y-6)-i(x-2)(y-6)+ixy}{x^2+(y-6)^2}$$

$$\frac{x^2-2x+y^2-6y}{x^2+(y-6)^2}+i\frac{(6x+2y-12)}{x^2+(y-6)^2}$$

Given that Amp $\left(\frac{z-2}{z-6i}\right)=\frac{\pi}{2}$

$$\tan^{-1}\left(\frac{6x+2y-12}{x^2+y^2-2x-6y}\right)=\frac{\pi}{2}$$

$$\frac{6x+2y-12}{x^2+y^2-2x-6y}=\tan\frac{\pi}{2} \quad\quad \because \text{Amp}z=\tan^{-1}\frac{y}{x}$$

$$\frac{6x+2y-12}{x^2+y^2-2x-6y}=\frac{1}{0}$$

$x^2+y^2-2x-6y=0$

6. **Determine the locus of a, $z \neq 2i$ such that** $\text{Re}\left[\frac{z-4}{z-2i}\right]=0$

Sol. Let z=x+iy

Now $\frac{z-4}{z-2i}=\frac{x+iy-4}{x+iy-2i}=\frac{(x-4)+iy}{x+i(y-2)}$

$$\frac{(x-4)+iy}{x+i(y-2)}\times\frac{x-i(y-2)}{x-i(y-2)}=\frac{[(x-4)+iy][x-i(y-2)]}{x^2-i^2(y-2)^2}$$

$$\frac{x(x-4)-i(x-4)(y-2)+iny-i^2y(y-2)}{x^2+(y-2)^2}$$

$$\frac{x(x-4)+y(y-2)}{x^2+(y-2)^2}-i\frac{[(x-4)(y-2)-xy]}{x^2+(y-2)^2}$$

Given that $\text{Re}\left(\frac{z-4}{z-2i}\right)=0$

$$\frac{x(x-4)+y(y-2)}{x^2+(y-2)^2}=0$$

$$\frac{x^2-4x+y^2-2y}{x^2+(y-2)^2}=0$$

$x^2+y^2-4x-2y=0$

$\therefore$ The locus o z represents a circle

7. **If $z=2-i\sqrt{7}$ then show that $3z^3-4z^2+z+88=0$**

Sol. $z=2-i\sqrt{7} \Rightarrow z-2=-i\sqrt{7}$

Squaring on both sides

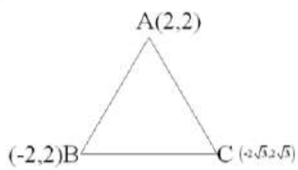
$(z-2)^2=\left(-i\sqrt{7}\right)^2 \Rightarrow (z-2)^2=7i^2 \Rightarrow (z-2)^2=-7$

$Z^2-4z+4=-7 \Rightarrow (z-2)^2=7i^2 \Rightarrow (z-2)^2=-7$

$3z^3-4z^2+z+88=3z(z^2-4z+11)+8(z^2-4z+11)=3z(0)+8(0)=0$

8. **Show that the points in the Argand plane represented by the complex numbers 2+2i, -2-2i, $-2\sqrt{3}+2\sqrt{3}i$ are the vertices of an equilateral triangle**

Sol. Given points in the Argand diagram are A(2,2), B(-2,-2), C$\left(-2\sqrt{3},2\right.$



$$AB=\sqrt{(2+2)^2+(2+2)^2}=\sqrt{4^2+4^2}=\sqrt{16+16}=\sqrt{32}$$

$$BC=\sqrt{\left(-2+2\sqrt{3}\right)^2+\left(-2-2\sqrt{3}\right)^2}=\sqrt{4+12-8\sqrt{3}+4+12+8\sqrt{3}}=\sqrt{16+16}=\sqrt{32}$$

$$CA=\sqrt{\left(2+2\sqrt{3}\right)^2+\left(2-2\sqrt{3}\right)^2}=\sqrt{4+12+8\sqrt{3}+4+12-8\sqrt{3}}=\sqrt{16+16}=\sqrt{32}$$

AB=BC=CA, In equilateral triangle all sides are equal

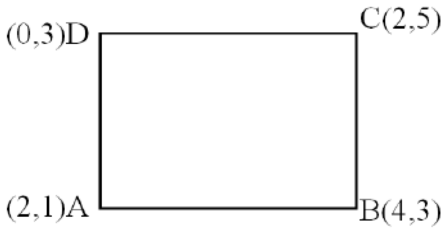
$\therefore$ The points A,B,C form an equilateral triangle

9. **Show that the four points in the Argand plane represented by the complex numbers 2+i,4+3i,2+5i,3i are the vertices of a square**

Sol. Given A=2+i=(2,1),B=4+3i, C=2+5i=(2,5), D=3i=(0,3)

$\because$ Distance between two points $A(x_1, y_1), B(x_2,y_2)$ is $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

AB=$\sqrt{(4-2)^2+(3-1)^2}=\sqrt{2^2+2^2}=\sqrt{4+4}=\sqrt{8}$



$BC=\sqrt{(2-4)^2+(5-3)^2}=\sqrt{(-2)^2+2^2}=\sqrt{4+4}=\sqrt{8}$

$CD=\sqrt{(0-2)^2+(3-5)^2}=\sqrt{(-2)^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}$

$DA=\sqrt{(2-0)^2+(1-3)^2}=\sqrt{2^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}$

$AC=\sqrt{(2-2)^2+(5-1)^2}=\sqrt{0+4^2}=\sqrt{16}=4$

$BD=\sqrt{(0-4)^2+(3-3)^2}=\sqrt{(-4)^2+0}=\sqrt{16}=4$

$\therefore$ AB=BC=CD=DA and AC=BD

$\because$ In the square all sides are equal and diagonals are also equal

$\therefore$ A,B,C,D Form a square

10. **Show that the points in the Argand plane represented by the complex numbers -2+7i, $\frac{-3}{2}+\frac{1}{2}i, 4-3i, \frac{7}{2}(1+i)$ are the vertices of a Rhombus**

Sol. Given points in the Argand plane are

$A=-2+7i=(-2,7), B=\frac{-3}{2}+\frac{1}{2}i=\left(\frac{-3}{2},\frac{1}{2}\right)$

$C=4-3i=(4,-3), d=\frac{7}{2}+\frac{7}{2}i=\left(\frac{7}{2},\frac{7}{2}\right)$

$\because$ Distance between two points $A(x_1,y_1)$ and $B(x_2,y_2)$ is $AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

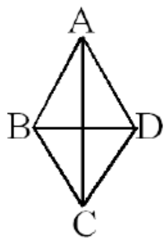
$AB=\sqrt{\left(\frac{-3}{2}+2\right)^2+\left(\frac{1}{2}-7\right)^2}=\sqrt{\left(\frac{1}{2}\right)2+\left(\frac{-13}{2}\right)2}=\sqrt{\frac{1}{4}+\frac{169}{4}}=\sqrt{\frac{170}{4}}$

$BC=\sqrt{\left(4+\frac{3}{2}\right)^2+\left(-3-\frac{1}{2}\right)^2}=\sqrt{\left(\frac{11}{2}\right)^2+\left(\frac{-7}{2}\right)^2}=\sqrt{\frac{121}{4}+\frac{49}{4}}=\sqrt{\frac{170}{4}}$

$CD=\sqrt{\left(\frac{7}{2}-4\right)^2+\left(\frac{7}{2}+3\right)^2}=\sqrt{\left(\frac{-1}{2}\right)^2+\left(\frac{13}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{169}{4}}=\sqrt{\frac{170}{4}}$

$DA=\sqrt{\left(-2-\frac{7}{2}\right)^2+\left(7-\frac{7}{2}\right)^2}=\sqrt{\left(\frac{-11}{2}\right)^2+\left(\frac{7}{2}\right)^2}=\sqrt{\frac{121}{4}+\frac{49}{4}}=\sqrt{\frac{170}{4}}$

$AC=\sqrt{(4+2)^2+(-3-7)^2}=\sqrt{6^2+(-10)^2}=\sqrt{36+100}=\sqrt{136}$



$BD=\sqrt{\left(\frac{7}{2}+\frac{3}{2}\right)^2+\left(\frac{7}{2}-\frac{1}{2}\right)^2}=\sqrt{\left(\frac{10}{2}\right)^2+\left(\frac{1}{2}\right)^2}=\sqrt{\frac{100}{4}+\frac{36}{4}}=\sqrt{\frac{136}{4}}$

$\therefore$ AB=BC=CD=DA and $AC \neq BD$

$\because$ In the rhombous all sides are equal but diagonals are not equal

11. **Show that $\frac{2-i}{(1-2i)^2}$ and $\left(\frac{-2-11i}{25}\right)$ are conjugate to each other**

Sol. Let $z_1=\frac{2-i}{(1-2i)^2}, z_2=\frac{-2-11i}{25}$

$z_1=\frac{2-i}{1+4i^2-4i}=\frac{2-i}{1-4-4i}=\frac{2-i}{-3-4i}\times\frac{-3+4i}{-3+4i}$

$$=\frac{-6+8i+3i-4i^2}{(-3)^2-(4i)^2}=\frac{-6+4+11i}{9-16i^2}$$

$$=\frac{-2+11i}{9+16}$$

$$=\frac{-2+11i}{25}$$

$\therefore$ z_1 is the conjugate of z_2

***Level-2*:**

1. **If $u+iv=\frac{2+i}{z+3}$ and z=x+iy then find u, v**

Sol. Given $u+iv=\frac{2+i}{z+3}=\frac{2+i}{x+iy+3}=\frac{2+i}{(x+3)+iy}$

$$=\frac{2+i}{(x+3)+iy}\times\frac{(x+3)-iy}{(x+3)-iy}$$

$$=\frac{(2+i)\left[(x+3)-iy\right]}{(x+3)^2-i^2y^2}$$

$$=\frac{2(x+3)-2iy+i(x+3)-i^2y}{(x+3)^2+y^2}$$

$$=\frac{2x+6+y+i(x+3-2y)}{(x+3)^2+y^2}$$

u+iv= $\frac{(2x+y+6)}{(x+3)^2+y^2}+\frac{i(x-2y+3)}{(x+3)^2+y^2}$

equating real and imaginary parts

$$u=\frac{2x+y+6}{(x+3)^2+y^2},v=\frac{x-2y+3}{(x+3)^2+y^2}$$

2. **The complex number z has argument 0, $0<\theta<\frac{\pi}{2}$ and satisfying the equation |z-3i|=3, then prove that $\left(\cot\theta-\frac{6}{z}\right)=i$**

Sol. Let z=x+iy

$\Rightarrow \theta=\tan^{-1}\frac{y}{x}\Rightarrow \tan\theta=\frac{y}{x}$ so $\cot\theta=\frac{x}{y}$

Given that |z-3i|=3

$\Rightarrow |x+iy-3i|=3\Rightarrow |x+i(y-3)|$

$\Rightarrow \sqrt{x^2+(y-3)^2}=3$ SOBS

$\Rightarrow x^2+(y-3)^2=9$

$\Rightarrow x^2+y^2-6y+9=9\Rightarrow x^2+y^2=6y$ ....(1)

Consider $\left(\cot\theta-\frac{6}{z}\right)=\frac{x}{y}-\frac{6}{x+iy}=\frac{x}{y}-\frac{6(x-iy)}{(x+iy)(x-iy)}$

$$=\frac{x}{y}-\frac{6(x-iy)}{x^2-i^2y^2}=\frac{x}{y}-\frac{6(x-iy)}{x^2+y^2}$$

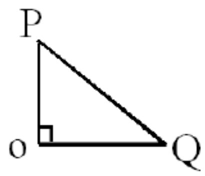
$$=\frac{x}{y}-\frac{6(x-iy)}{6y} \quad \text{From (1)}$$

$$=\frac{x}{y}-\frac{x}{y}+\frac{iy}{y}=i$$

3. **The points P, Q denote the complex numbers z_1,z_2 in the Argand diagram, O is the origin. If $z_1\overline{z_2}+z_1\overline{z_2}=0$ Then show that $\angle POQ=90^0$**

Sol. Let $z_1=x_1+iy_1$ and $z_2=x_2+iy_2$ then $\overline{z_1}=x_1-iy_1$ and $\overline{z_2}=x_2-iy_2$

The points z_1,z_2 in the Argand diagram are $P(x_1,y_1)$ $Q(x_2,y_2)$ and (0,0), slope of $OP=\frac{y_1}{x_1}$, slope $OQ=\frac{y_2}{x_2}$



$$z_1\overline{z_2}+\overline{z_1}z_2=0 \Rightarrow (x_1+iy_1)(x_2-iy_2)+(x_1-iy_1)(x_2+iy_2)=0$$

$$\Rightarrow x_1x_2-i\frac{x}{1}y_2+i\frac{x}{2}y_1-i^2y_1y_2+x_1x_2+i\frac{x}{1}y_2-i\frac{x}{2}y_1-i^2y_1y_2=0$$

$$\Rightarrow 2x_1x_2+2y_1y_2=0 \Rightarrow x_1x_2+y_1y_2=0$$

$$\Rightarrow y_1y_2=-x_1x_2 \Rightarrow \left(\frac{y_1}{x_1}\right)\left(\frac{y_2}{x_2}\right)=-1$$

$\Rightarrow$(slope of $\overline{OP}$) (slope of $\overline{OQ}$)=-1 $\Rightarrow \angle POQ=90^0$

4. **If $\frac{z_2}{z_1},z_1\neq 0$ is an imaginary number then find the value of $\left|\frac{2z_1+z_2}{2z_1-z_2}\right|$**

Sol. $\frac{z_2}{z_1},(z_1\neq 0)$ is purely imaginary

we can suppose that $\frac{z_2}{z_1}=iy$

$$\left|\frac{2z_1+z_2}{2z_1-z_2}\right|=\left|\frac{z_1\left(2+\frac{z_2}{z_1}\right)}{z_1\left(2-\frac{z_2}{z_1}\right)}\right|$$

$$=\left|\frac{2+\frac{z_2}{z_1}}{2-\frac{z_2}{z_1}}\right|$$

$$=\left|\frac{2+iy}{2-iy}\right|$$

$$=\frac{\sqrt{4+y^2}}{\sqrt{4+y^2}} \qquad \because |x+iy|=\sqrt{x^2+y^2}$$

$$\therefore \left|\frac{2z_1+z_2}{2z_1+z_2}\right|=1$$

Chapter-2

De Moivre's Theorem

Weightage : (2 + 7)

Key Concepts:

$\rightarrow$ De Moivre's Theorem: If 'n' is an integer and 'θ' be any real number then

(i) $(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$

(ii) $(\cos\theta-i\sin\theta)^n=\cos n\theta-i\sin n\theta$

If x = $\cos\theta+i\sin\theta$ then $\frac{1}{x}=\cos\theta-i\sin\theta$ and

(i) $x+\frac{1}{x}=2\cos\theta$ (ii) $x-\frac{1}{x}=2i\sin\theta$ (iii) $x^n+\frac{1}{x^n}=2\cos n\theta$ (iv) $x^n-\frac{1}{x^n}=2i\sin n\theta$

Here $\cos\theta+i\sin\theta=cis\theta$, $\cos\theta - i\sin\theta = cis(-\theta)$

Cube roots of unity

The roots of $x^3=1$ are called cube roots of unity then which are $1,w,w^2$ where $w=\frac{-1+i\sqrt{3}}{2}, w^2=\frac{-1-i\sqrt{3}}{2}$

If $1,w,w^2$ are the cube roots of unity then

(i) $1+w+w^2=0 \Rightarrow 1+w=-w^2 \Rightarrow 1+w^2=-w \Rightarrow w+w^2=-1$

(ii) $w^3=1, w^4=w^3.w=w, w^5=w^3.w^2=w^2, w^6=(w^3)^2=1$

n^{th} roots of a complex number

The n^{th} roots of a complex number

$z=r(\cos\theta+i\sin\theta)$ are

$z^{\frac{1}{n}}=r^{\frac{1}{n}}cis\left(\frac{2k\pi+\theta}{n}\right)$; where k=0, 1,2 ……. (n-1)

$cis\theta.cis\phi=cis(\theta+\phi)$ for any $\theta,\phi\in R$

$\frac{cis\theta}{cis\phi}=cis(\theta-\phi)$ for any $\theta,\phi\in R$

Level-1:

Very Short Answer Questions (2Marks)

1. **If A, B, C are the angles of a triangle and x=cisA, y=cisB, z=cisC find the value of xyz**

Sol. Given x=cisA, y=cisB, z=cisC

in ΔABC, $A+B+C=180^0$

x.y.z=cisA.cisB.cisC

$= cis(A+B+C)$ $\because cis\theta . cis\phi = cis(\theta+\phi)$

$= cis180^0$ $\because cis\theta = cos\theta + isin\theta$

$= cos180^0 + isin180^0$

$= -1$

2. **If** $x = cis\theta$ **then find the value of** $\left(x^6 + \frac{1}{x^6}\right)$

Sol. $x = cis\theta = cos\theta + isin\theta$

$x^6 = (cis\theta)^6 = cos6\theta + isin6\theta$

$\frac{1}{x^6} = cos6\theta - isin6\theta$ $\because (cos\theta + isin\theta)^n = cosn\theta + isinn\theta$

$\therefore x^6 + \frac{1}{x^6} = cos6\theta + isin6\theta + cos6\theta - isin6\theta$

$= 2cos6\theta$

3. **Find the cube roots of '8'?**

Sol. $\sqrt[3]{8} = 8^{\frac{1}{3}} = ((8)(1))^{\frac{1}{3}} = (8)^{\frac{1}{3}}.(1)^{\frac{1}{3}} = (2^3)^{\frac{1}{3}}(1)^{\frac{1}{3}} = 2(1)^{\frac{1}{3}}$

$= 2(1,w,w^2) = 2(1), 2(w), 2(w)$ Cube roots of '1' are 1, w, w^2

$\therefore$ Cube roots of '8' are 2,2w, $2w^2$

4. **Find the roots of the equation** $(x-1)^3 + 8 = 0$**. If the cube roots of unity are 1,w,w^2**

Sol. $(x-1)^3 + 8 = 0 \Rightarrow (x-1)^3 = -8 = -2^3$

$= (x-1)^3 = -2^3 \Rightarrow x-1 = -2 \Rightarrow x-1 = -2(1)^{\frac{1}{3}} = -2(1,w,w^2)$

$\therefore$ The roots of x-1 are -2,-2w, $-2w^2$

Hence the roots of x are -1, 1-2w, $1-2w^2$

5. **Find the value of** $(1+i\sqrt{3})^3$

Sol. $1+i\sqrt{3} = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$ $\because$ multiplying and dividing by $\sqrt{a^2+b^2} = \sqrt{1+3} = 2$

$= 2\left[cos\frac{\pi}{3} + isin\frac{\pi}{3}\right]$

$(1+i\sqrt{3})^3 = \left[2\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)\right]^3$

$= 2^3\left(cos\frac{\pi}{3} + isin\frac{\pi}{3}\right)^3$

$= 8\left(cos3\frac{\pi}{3} + isin3\frac{\pi}{3}\right)$

$= 8(cos\pi + isin\pi)$

$= 8(-1) = -8$

6. **Find the value of $(1-i)^8$**

Sol. $(1-i) = \sqrt{2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)$ = $\because$ multiplying & Dividing by $\sqrt{a^2+b^2} = \sqrt{1^2+(-1)^2} = \sqrt{2}$

$$=\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)$$

$$(1-i)^8=\left[\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)\right]^8$$

$$=\left(\sqrt{2}\right)^8\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)^8$$

$$=2^4\left(\cos8\frac{\pi}{4}-i\sin8\frac{\pi}{4}\right)$$

$$=2^4\left(\cos2\pi-i\sin2\pi\right)$$

=16(1-i(0))=16(1)=16

7. **Find the value of** $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5-\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$

Sol. $\left(\frac{\sqrt{3}}{2}+i\left(\frac{1}{2}\right)\right)=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$

$\left(\frac{\sqrt{3}}{2}-i\left(\frac{1}{2}\right)\right)=\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}$

$$\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5-\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5=\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^5-\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^5$$

$$=\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)-\left(\cos\frac{5\pi}{6}-i\sin\frac{5\pi}{6}\right)$$

$$=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}-\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}$$

$$=2i\sin\frac{5\pi}{6}$$

$$=2i\sin\left(\pi-\frac{\pi}{6}\right)=2i\sin\frac{\pi}{6}$$

$$=2i\left(\frac{1}{2}\right)=i$$

8. **Find all the values of** $\left(\sqrt{3}+i\right)^{\frac{1}{4}}$

Sol. $\sqrt{3}+i=2\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)=2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ $\quad\because$ Multiplying & Dividing by $=\sqrt{a^2+b^2}=\sqrt{1+3}=2$

$$=\sqrt{3}+i=2\left[\cos\left(2k\pi+\frac{\pi}{6}\right)+i\sin\left(2k\pi+\frac{\pi}{6}\right)\right]$$

$$=2\left[\operatorname{cis}\left(2k\pi+\frac{\pi}{6}\right)\right]$$

$$\left(\sqrt{3}+i\right)^{\frac{1}{4}}=\left[2\left(\operatorname{cis}\left(2k\pi+\frac{\pi}{6}\right)\right)\right]^{\frac{1}{4}}$$

$$=2^{\frac{1}{4}}\left[\text{cis}\left(2k\pi+\frac{\pi}{6}\right)\right]^{\frac{1}{4}}$$

$$=2^{\frac{1}{4}}\left[\text{cis}\left(\frac{12k\pi+\pi}{6}\right)\right]^{\frac{1}{4}}$$

$$=2^{\frac{1}{4}}\text{cis}(12k+1)\frac{\pi}{24},k=0,1,2,3$$

9. **Find all the values of** $(-i)^{\frac{1}{6}}$

Sol. $-i=\cos\left(\frac{-\pi}{2}\right)+i\sin\left(\frac{-\pi}{2}\right)$

$$=\cos\left(2k\pi-\frac{\pi}{2}\right)+i\sin\left(2k\pi-\frac{\pi}{2}\right)$$

$$=\cos\left(\frac{4k\pi-\pi}{2}\right)+i\sin\left(\frac{4k\pi-\pi}{2}\right)$$

$$=\cos\frac{(4k-1)\pi}{2}+i\sin\frac{(4k-1)\pi}{2}$$

$$(-i)^{\frac{1}{6}}=\left[\cos\frac{(4k-1)\pi}{2}+i\sin\frac{(4k-1)\pi}{2}\right]^{\frac{1}{6}}$$

$$=\text{cis}\frac{(4k-1)\pi}{2}.\frac{1}{6}$$

$=\text{cis}\frac{(4k-1)\pi}{12}$, where k=0,1,2,3,4,5

10. **Find all the values of** $(-32)^{\frac{1}{5}}$

Sol. -32=32(-1)= $2^5(\cos\pi+i\sin\pi)$

$$= 2^5(\cos(2k+1)\pi+i\sin(2k+1)\pi)$$

$$(-32)^{\frac{1}{5}}=(2^5)^{\frac{1}{5}}\left[\cos(2k+1)\pi+i\sin(2k+1)\pi\right]^{\frac{1}{5}}$$

$$=2\text{cis}(2k+1)\pi.\frac{1}{5}$$

$=2\text{cis}\frac{(2k+1)\pi}{5}$, k=0,1,2,3,4

11. **If 1,w,w^2 are the cube roots of unity then find the values of (i) $(1+w+w^2)^3$ (ii) $(1-w)(1-w^2)(1-w^4)(1-w^8)$ (iii)** $(1-w+w^2)^5+(1+w-w^2)^5$ **(iv)** $\left(\frac{a+bw+cw^2}{c+aw+bw^2}\right)+\left(\frac{a+bw+cw^2}{b+cw+aw^2}\right)$

Sol. (i) $(1-w+w^2)^3=(-w-w)^3=(-2w)^3=-8w^3=-8(1)=-8$

$$\because 1+w+w^2=0 \Rightarrow 1+w^2=-w$$

$$w^3=1$$

(ii) $(1-w)(1-w^2)(1-w^4)(1-w^8)$

$=(1-w)(1-w^2)(1-w)(1-w^2)$ $\qquad \because w^4=w^3.w=1.w=w$, $w^8=w^4.w^4=w.w=w^2$

$=[(1-w)(1-w^2)]^2$

$=(1-w-w^2+w^3)^2$ $\qquad \because 1+w+w^2=0 \Rightarrow -w-w^2=1 \Rightarrow w^3=1$

$=(1+1+1)^2=3^2=9$

(iii) $(1-w+w^2)^5+(1+w-w^2)^5$

$=(1+w^2-w)^5+(1+w-w^2)^5$

$=(-w-w)^5+(-w^2-w^2)^5$

$=(-2w)^5+(-2w^2)^5$

$=(-2)^5(w^5+w^{10})$ $\qquad \because 1+w+w^2=0$, $w^3=1$

$=-32(w^3.w^2+(w^3)^3.w)$

$=-32(w^2+w)=-32(-1)=32$

(iv) $\left(\frac{a+bw+cw^2}{c+aw+bw^2}\right)+\left(\frac{a+bw+cw^2}{b+cw+aw^2}\right)$

$=\frac{w^2\left(a+bw+cw^2\right)}{w^2\left(c+aw+bw^2\right)}+\frac{w\left(a+bw+cw^2\right)}{w\left(b+cw+aw^2\right)}$

$=\frac{w^2\left(a+bw+cw^2\right)}{aw^3+bw^4+cw^2}+\frac{w\left(a+bw+cw^2\right)}{aw^3+cw^2+bw}$

$=\frac{w^2\left(a+bw+cw^2\right)}{a+bw+cw^2}+\frac{w\left(a+bw+cw^2\right)}{a+bw+cw^2}$

$=w^2+w=-1$ $\qquad \because 1+w+w^2=0 \Rightarrow w^2+w=-1 \Rightarrow w^4=w$

12. **If 1,w,w^2 are the cube roots of unity, then prove that**

(i) $\frac{1}{2+w}+\frac{1}{1+2w}=\frac{1}{1+w}$ **(ii)** $(2-w)\left(2-w^2\right)\left(2-w^{10}\right)\left(2-w^{11}\right)=49$

(iii) $(x+y+z)\left(x+yw+zw^2\right)\left(x+yw^2+zw\right)=x^3+y^3+z^3-3xyz$

Sol. (i) $\frac{1}{2+w}+\frac{1}{1+2w}=\frac{1+2w+2+w}{(2+w)(1+2w)}$

$=\frac{3+3w}{(2+w)(1+2w)}$

$=\frac{3(1+w)}{2+4w+w+2w^2}$

$=\frac{3\left(-w^2\right)}{2+2w+2w^2+3w}$

$=\frac{-3w^2}{2\left(1+w+w^2\right)+3w}$

$=\frac{-3w^2}{3w}$

$=-w$

$=\frac{-w(1+w)}{1+w}$

$$=\frac{-w-w^2}{1+w}$$
$$=\frac{1}{1+w}$$
$$\therefore \frac{1}{2+w}+\frac{1}{1+2w}=\frac{1}{1+w}$$

(ii) $(2-w)(2-w^2)(2-w^{10})(2-w^{11})$

$$=(2-w)(2-w^2)\left(2-(w^3)^3.w\right)\left(2-(w^3)^3.w^2\right)$$

$\because w^{10}=(w^3)^3.w=1.w=w$

$w^{11}=(w^3)^3.w^2=1.w^2=w^2$

$$=(2-w)(2-w^2)(2-w)(2-w^2)$$
$$=\left[(2-w)(2-w^2)\right]^2$$
$$=\left[4-2w-2w^2+w^3\right]^2$$
$$=\left[4-2(w+w^2)+w^3\right]^2$$

$\because 1+w+w^2=0 \Rightarrow w+w^2=-1$

$w^3=1$

$$=\left[4-2(-1)+1\right]^2$$
$$=[4+2+1]^2$$
$$=7^2$$
$$=49$$
$$\therefore (2-w)(2-w^2)(2-w^{10})(2-w^{11})=49$$

(iii) $(x+y+z)(x+yw+zw^2)(x+yw^2+zw)$

$$=(x+y+z)\left[x^2+xyw^2+xzw+xyw+y^2w^3+yzw^2+xzw^2+yzw^4+z^2w^3\right]$$
$$=(x+y+z)\left[x^2+y^2+z^2+xy(w^2+w)+yz(w^2+w^4)+zx(w+w^2)\right]$$
$$=(x+y+z)\left[x^2+y^2+z^2+xy(-1)+yz(-1)+zx(-1)\right]$$
$$=(x+y+z)\left[x^2+y^2+z^2-xy-yz-zx\right]$$
$$=x^3+y^3+z^3-3xyz$$
$$\therefore (x+y+z)(x+yw+zw^2)(x+yw^2+zw)=x^3+y^3+z^3-3xyz$$

13. Prove that $(a+b)(aw+bw^2)(aw^2+bw)=a^3+b^3$. If $1,w,w^2$ are the cube roots of unity

Sol. $(a+b)(aw+bw^2)(aw^2+bw)$

$$=(a+b)\left[a^2w^3+abw^2+abw^4+b^2w^3\right]$$
$$=(a+b)\left[a^2(1)+abw^2+ab(w)+b^2(1)\right]$$
$$=(a+b)\left[a^2+ab(w^2+w)+b^2\right]$$
$$=(a+b)\left[a^2+ab(-1)+b^2\right]$$
$$=(a+b)\left[a^2-ab+b^2\right]$$
$$=a^3+b^3$$
$$\therefore (a+b)(aw+bw^2)(aw^2+bw)=a^3+b^3$$

14. **Solve x^4-1=0**

Sol. $x^4-1=0 \Rightarrow (x^2+1)(x^2-1)=0$

$\Rightarrow x^2+1=0 \text{ (or) } x^2-1=0$

$\Rightarrow x^2=-1 \text{ (or) } x^2=1$

$\Rightarrow x=\sqrt{-1} \text{ or } x=\sqrt{1}$

$\Rightarrow x=\pm i \text{ or } x=\pm 1$

15. **Simplify** $\dfrac{(\cos\alpha+i\sin\alpha)^4}{(\sin\beta+i\cos\beta)^8}$

Sol. $\dfrac{(\cos\alpha+i\sin\alpha)^4}{(\sin\beta+i\cos\beta)^8}=\dfrac{(\cos\alpha+i\sin\alpha)^4}{(-i^2\sin\beta+i\cos\beta)^8}$

$=\dfrac{(\cos\alpha+i\sin\alpha)^4}{[i(\cos\beta-i\sin\beta)]^8}$

$=\dfrac{(\cos\alpha+i\sin\alpha)^4}{i^8(\cos\beta-i\sin\beta)^8}$

$=\dfrac{\cos4\alpha+i\sin4\alpha}{\cos8\beta-i\sin8\beta}$

$=(\cos 4\alpha+i\sin 4\alpha)(\cos 8\beta+i\sin 8\beta)$

$=\cos(4\alpha+8\beta)+i\sin(4\alpha+8\beta)$

16. **If** α,β **are the roots of the equation** $x^2+x+1=0$**, then prove that** $\alpha^4+\beta^4+\alpha^{-1}\beta^{-1}=0$

Sol. $x^2+x+1=0$

Since α,β are the complex cube roots of unity take $\alpha=w,\beta=w^2$

$\therefore \alpha^4+\beta^4+\alpha^{-1}\beta^{-1}=w^4+(w^2)^4+(w)^{-1}(w^2)^{-1}$

$=w^3.w+(w^3)^2.w^2+\dfrac{1}{w^3}$ $\quad\because 1+w+w^2=0$, $w^3=1$

$=w+w^2+1=0$

17. **If 1,w,w^2 are the cube roots of unity, then find the value of**

(i) $(a+2b)^2+(aw^2+2bw)^2+(aw+2bw^2)^2$ **(ii)** $(1+w)^3+(1+w^2)^3$

Sol. (i) $(a+2b)^2+(aw^2+2bw)^2+(aw+2bw^2)^2$

$(a+2b)^2=a^2+4ab+4b^2$ .... (1)

$(aw^2+2bw)^2=a^2w^4+4abw^3+4b^2w^2=a^2w^3.w+4abw^3+4b^2w^2$

$=a^2w+4ab+4b^2w^2$..................... (2)

$(aw+2bw^2)^2=a^2w^2+4abw^3+4b^2w^4$ $\quad\because 1+w+w^2=0$, $w^3=1$

$=a^2w^2+4ab+4b^2w$ .... (3)

By adding (1), (2), & (3)

$(a+2b)^2+(aw^2+2bw)^2+(aw+2bw^2)^2$

$=a^2+4ab+4b^2+a^2w+4ab+4b^2w^2+a^2w^2+4ab+4b^2w$

$=a^2(1+w+w^2)+12ab+4b^2(1+w+w^2)$

$=a^2(0)+12ab+4b^2(0)$

=12ab

(ii) $(1+w)^3+(1+w^2)^3$

$=(-w^2)^3+(-w)^3$

$=-w^6-w^3$ $\quad \because 1+w+w^2=0$, $w^3=1$

=-1-1

=-2

Long Answer questions(7Marks)

Level-1 :

1. **If 'n' is an integer, then show that** $(1+i)^{2n}+(1-i)^{2n}=2^{n+1}\cos\frac{n\pi}{2}$

Sol. $1+i=\sqrt{2}\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)$ Multiply and Divide with $\sqrt{a^2+b^2}=\sqrt{1+1}=\sqrt{2}$

$=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$(1+i)^{2n}=\left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^{2n}$

$=\left(\sqrt{2}\right)^{2n}\left(\cos\frac{2n\pi}{4}+i\sin\frac{2n\pi}{4}\right)$ $\quad \because (\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$

$=2^n\left(\cos\frac{n\pi}{2}+i\sin\frac{n\pi}{2}\right)$ .... (1)

$1-i=\sqrt{2}\left(\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}\right)$

$=\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)$

$(1-i)^{2n}=\left[\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)\right]^{2n}$

$=\left(\sqrt{2}\right)^{2n}\left(\cos\frac{2n\pi}{4}-i\sin\frac{2n\pi}{4}\right)$

$=2^n\left(\cos\frac{n\pi}{2}-i\sin\frac{n\pi}{2}\right)$ .... (2)

Adding (1) & (2)

$(1+i)^{2n}+(1-i)^{2n}=2^n\left(\cos\frac{n\pi}{2}+i\sin\frac{n\pi}{2}\right)+2^n\left(\cos\frac{n\pi}{2}-i\sin\frac{n\pi}{2}\right)$

$=2^n\left[\cos\frac{n\pi}{2}+i\sin\frac{n\pi}{2}+\cos\frac{n\pi}{2}-i\sin\frac{n\pi}{2}\right]$

$=2^n.2\cos\frac{n\pi}{2}$

$$=2^{n+1}\cos\frac{n\pi}{2}$$

$$\therefore (1+i)^{2n}+(1-i)^{2n}=2^{n+1}\cos\frac{n\pi}{2}$$

2. **If 'n' is an integer then show that** $(1+i)^{n}+(1-i)^{n}=2^{\frac{n+2}{2}}\cos\frac{n\pi}{4}$

Sol. $1+i=\sqrt{2}\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)$

$$=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$$

$$(1+i)^{n}=\left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^{n}$$

$$=\left(\sqrt{2}\right)^{n}\left(\cos\frac{n\pi}{4}+i\sin\frac{n\pi}{4}\right)$$

$$=2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4}+i\sin\frac{n\pi}{4}\right)\text{..................(1)}$$

$$1-i=\sqrt{2}\left(\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}\right)$$

$$=\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)$$

$$(1-i)^{n}=\left[\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)\right]^{n}$$

$$=\left(\sqrt{2}\right)^{n}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)^{n}$$

$$=2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4}-i\sin\frac{n\pi}{4}\right)\text{.............. (2)}$$

Adding (1) & (2)

$$(1+i)^{n}+(1-i)^{n}=2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4}+i\sin\frac{n\pi}{4}\right)+2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4}-i\sin\frac{n\pi}{4}\right)$$

$$=2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4}+i\sin\frac{n\pi}{4}+\cos\frac{n\pi}{4}-i\sin\frac{n\pi}{4}\right)$$

$$=2^{\frac{n}{2}}.2\cos\frac{n\pi}{4}$$

$$=2^{\frac{n+2}{2}}\cos\frac{n\pi}{4}$$

$$\therefore (1+i)^{n}+(1-i)^{n}=2^{\frac{n+2}{2}}\cos\frac{n\pi}{4}$$

3. **If** α,β **are the roots of the equation** $x^2-2x+4=0$**, then show that** $\alpha^{n}+\beta^{n}=2^{n+1}\cos\frac{n\pi}{3}$

Sol. $x^2-2x+4=0\Rightarrow x=\frac{2\pm\sqrt{4-16}}{2}=\frac{2\pm\sqrt{-12}}{2}=\frac{2\pm2\sqrt{-3}}{2}=\frac{2\pm2\sqrt{3i^2}}{2}=\frac{2\left(1\pm\sqrt{3}i\right)}{2}=1\pm\sqrt{3}i$

Let $\alpha=1+\sqrt{3}i,\ \beta=1-\sqrt{3}i$

$$\alpha^n+\beta^n=\left(1+\sqrt{3}i\right)^n+\left(1-\sqrt{3}i\right)^n$$

$$=\left[2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\right]^n+\left[2\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)\right]^n$$

$$=\left[2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]^n+\left[2\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)\right]^n$$

$$=2^n\left(\cos\frac{n\pi}{3}+i\sin\frac{n\pi}{3}\right)+2^n\left(\cos\frac{n\pi}{3}-i\sin\frac{n\pi}{3}\right)$$

$$=2^n\left[\cos\frac{n\pi}{3}+i\sin\frac{n\pi}{3}+\cos\frac{n\pi}{3}-i\sin\frac{n\pi}{3}\right]$$

$$=2^n.2\cos\frac{n\pi}{3}$$

$$=2^{n+1}\cos\frac{n\pi}{3}$$

$$\therefore \alpha^n+\beta^n=2^{n+1}\cos\frac{n\pi}{3}$$

4. **If** $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ **then show that**

(i) $\cos3\alpha+\cos3\beta+\cos3\gamma=3\cos(\alpha+\beta+\gamma)$

(ii) $\sin3\alpha+\sin3\beta+\sin3\gamma=3\sin(\alpha+\beta+\gamma)$

(iii) $\cos(\alpha+\beta)+\cos(\beta+\gamma)+\cos(\gamma+\alpha)=0$

Sol. Let $a=\cos\alpha+i\sin\alpha, b=\cos\beta+i\sin\beta, c=\cos\gamma+i\sin\gamma$

$a+b+c=\cos\alpha+i\sin\alpha+\cos\beta+i\sin\beta+\cos\gamma+i\sin\gamma$

$=(\cos\alpha+\cos\beta+\cos\gamma)+i(\sin\alpha+\sin\beta+\sin\gamma)$

=0+i(0)

=0

$\therefore a+b+c=0 \Rightarrow a^3+b^3+c^3=3abc$

$\Rightarrow (\cos\alpha+i\sin\alpha)^3+(\cos\beta+i\sin\beta)^3+(\cos\gamma+i\sin\gamma)^3=3(\cos\alpha+i\sin\alpha)(\cos\beta+i\sin\beta)(\cos\gamma+i\sin\gamma)$

$\Rightarrow \cos3\alpha+\cos3\beta+\cos3\gamma+i(\sin3\alpha+\sin3\beta+\sin3\gamma)=3\cos(\alpha+\beta+\gamma)+3i\sin(\alpha+\beta+\gamma)$

equating real and imaginary parts on both sides

(i) $\cos3\alpha+\cos3\beta+\cos3\gamma=3\cos(\alpha+\beta+\gamma)$

(ii) $\sin3\alpha+\sin3\beta+\sin3\gamma=3\sin(\alpha+\beta+\gamma)$

(iii) $a = cis\alpha, b = cis\beta, c = cis\gamma$

$\frac{1}{a}=cis(-\alpha), \frac{1}{b}=cis(-\beta), \frac{1}{c}=cis(-\gamma)$

$a+b+c=cis\alpha+cis\beta+cis\gamma=0$

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=cis(-\alpha)+cis(-\beta)+cis(-\gamma)$

$=\cos\alpha-i\sin\alpha+\cos\beta-i\sin\beta+\cos\gamma-i\sin\gamma$

$=\cos\alpha+\cos\beta+\cos\gamma-i(\sin\alpha+\sin\beta+\sin\gamma)$

$=0-i(0)=0$

Consider ab+bc+ca= $abc\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right)$

=abc(0)=0

ab+bc+ca=0

$\Rightarrow cis\alpha.cos\beta+cis\beta cis\gamma+cis\alpha.cis\gamma=0$

$\Rightarrow cis(\alpha+\beta)+cis(\beta+\gamma)+cis(\gamma+\alpha)=0$

$\Rightarrow \cos(\alpha+\beta)+i\sin(\alpha+\beta)+\cos(\beta+\gamma)+i\sin(\beta+\gamma)+\cos(\gamma+\alpha)+i\sin(\gamma+\alpha)=0=0+i(0)$

$\Rightarrow \cos(\alpha+\beta)+\cos(\beta+\gamma)+\cos(\gamma+\alpha)+i\left[\sin(\alpha+\beta)+\sin(\beta+\gamma)+\sin(\gamma+\alpha)\right]=0+i(0)$

Comparing the real part

$\cos(\alpha+\beta)+\cos(\beta+\gamma)+\cos(\gamma+\alpha)=0$

5. If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ **, then prove that** $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{3}{2}=\sin^2\alpha+\sin^2\beta+\sin^2\gamma$

Sol. Let x = $x=\cos\alpha+i\sin\alpha,\ y=\cos\beta+i\sin\beta,\ z=\cos\gamma+i\sin\gamma$

$x+y+z=\cos\alpha+i\sin\alpha+\cos\beta+i\sin\beta+\cos\gamma+i\sin\gamma$

$=\cos\alpha+\cos\beta+\cos\gamma+i(\sin\alpha+\sin\beta+\sin\gamma)$

$=0+i(0)$

$=0$

If $x+y+z=0\Rightarrow x^2+y^2+z^2=-2(xy+yz+zx)$

$x^2+y^2+z^2=-2xyz\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

$=-2xyz(\cos\alpha-i\sin\alpha+\cos\beta-i\sin\beta+\cos\gamma-i\sin\gamma)$

$=-2xyz(\cos\alpha+\cos\beta+\cos\gamma-i(\sin\alpha+\sin\beta+\sin\gamma))$

$=-2xyz\left[0-i(0)\right]$

$=-2xyz(0)$

=0

$\therefore x^2+y^2+z^2=0$

$\Rightarrow (\cos\alpha+i\sin\alpha)^2+(\cos\beta+i\sin\beta)^2+(\cos\gamma+i\sin\gamma)^2=0$

$\Rightarrow \cos2\alpha+i\sin2\alpha+\cos2\beta+i\sin2\beta+\cos2\gamma+i\sin2\gamma=0$

$\Rightarrow (\cos2\alpha+\cos2\beta+\cos2\gamma)+i(\sin2\alpha+\sin2\beta+\sin2\gamma)=0+i(0)$

Comparing real & imaginary

$\cos2\alpha+\cos2\beta+\cos2\gamma=0$

$\Rightarrow 2\cos^2\alpha-1+2\cos^2\beta-1+2\cos^2\gamma-1=0$

$\Rightarrow 2\cos^2\alpha+2\cos^2\beta+2\cos^2\gamma-3=0$

$\Rightarrow 2(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)=3$

$\Rightarrow \cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{3}{2}$

$1-\sin^2\alpha+1-\sin^2\beta+1-\sin^2\gamma=\frac{3}{2}$

$$3-\left(\sin^2\alpha+\sin^2\beta+\sin^2\gamma\right)=\frac{3}{2}$$

$$\sin^2\alpha+\sin^2\beta+\sin^2\gamma=3-\frac{3}{2}=\frac{3}{2}$$

6. **If n is an integer then show that** $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$

Sol. $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=\left(2\cos^2\frac{\theta}{2}+i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^n+\left(2\cos^2\frac{\theta}{2}-i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^n$

$$=\left[2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)\right]^n+\left[2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right)\right]^n$$

$$=\left(2\cos\frac{\theta}{2}\right)^n\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^n+\left(2\cos\frac{\theta}{2}\right)^n\left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right)^n$$

$$=2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}\right)+2^n\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}-i\sin\frac{n\theta}{2}\right)$$

$$=2^n\cos^n\frac{\theta}{2}\left[\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}+\cos\frac{n\theta}{2}-i\sin\frac{n\theta}{2}\right]$$

$$=2^n\cos^n\frac{\theta}{2}.2\cos\frac{n\theta}{2}$$

$$=2^{n+1}\cos^n\left(\frac{\theta}{2}\right).\cos\left(\frac{n\theta}{2}\right)$$

$$\therefore(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\frac{n\theta}{2}$$

7. **If 'n' is a positive integer then show that** $(P+iQ)^{\frac{1}{n}}+(P-iQ)^{\frac{1}{n}}=2\left(P^2+Q^2\right)^{\frac{1}{2n}}\cos\left(\frac{1}{n}\tan^{-1}\frac{Q}{P}\right)$

Sol. Let $P+iQ=\sqrt{P^2+Q^2}\left(\frac{P}{\sqrt{P^2+Q^2}}+i\frac{Q}{\sqrt{P^2+Q^2}}\right)$

$$P+iQ=r\left(\cos\theta+i\sin\theta\right)$$

$$\cos\theta=\frac{P}{\sqrt{P^2+Q^2}},\sin\theta=\frac{Q}{\sqrt{P^2+Q^2}}$$

$$r=\sqrt{P^2+Q^2},\tan\theta=\frac{Q}{P}$$

$(P+iQ)^{\frac{1}{n}}=r^{\frac{1}{n}}(\cos\theta+i\sin\theta)^{\frac{1}{n}}$ $\quad\quad$ $\because(\cos\theta+i\sin\theta)^n=\cos n\theta+i\sin n\theta$

$$=r^{\frac{1}{n}}\left(\cos\frac{\theta}{n}+i\sin\frac{\theta}{n}\right)$$

$$P-iQ=r\left(\cos\theta-i\sin\theta\right)$$

$$(P-iQ)^{\frac{1}{n}}=r^{\frac{1}{n}}(\cos\theta-i\sin\theta)^{\frac{1}{n}}$$

$$=r^{\frac{1}{n}}\left(\cos\frac{\theta}{n}-i\sin\frac{\theta}{n}\right)$$

$$\left(P+iQ\right)^{\frac{1}{n}}+\left(P-iQ\right)^{\frac{1}{n}}=r^{\frac{1}{n}}\left(\cos\frac{\theta}{n}+i\sin\frac{\theta}{n}\right)+r^{\frac{1}{n}}\left(\cos\frac{\theta}{n}-i\sin\frac{n\theta}{n}\right)$$

$$=r^{\frac{1}{n}}\left[\cos\frac{\theta}{n}+i\sin\frac{\theta}{n}+\cos\frac{\theta}{n}-i\sin\frac{\theta}{n}\right]$$

$$=r^{\frac{1}{n}}.2\cos^{\frac{\theta}{n}}$$

$$=2\left(\sqrt{P^2+Q^2}\right)^{\frac{1}{n}}\cos\left(\frac{1}{n}\tan^{-1}\frac{Q}{P}\right)$$

$$=2\left(P^2+Q^2\right)^{\frac{1}{2n}}\cos\left(\frac{1}{n}\ \text{Arc}\tan\frac{\theta}{P}\right) \qquad \because r=\sqrt{P^2+Q^2},\ \tan\theta=\frac{Q}{P}$$

$$\therefore \left(P+iQ\right)^{\frac{1}{n}}+\left(P-iQ\right)^{\frac{1}{n}}=2\left(P^2+Q^2\right)^{\frac{1}{2n}}\cos\left(\frac{1}{n}\ \text{Arc}\tan\frac{Q}{P}\right)$$

8. **Show that one value of** $\left[\dfrac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right]^{\frac{8}{3}}=-1$

Sol. $$\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}=\frac{1+\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)+i\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)}{1+\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)-i\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)}$$

$$\Rightarrow \frac{1+\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}}{1+\cos\frac{3\pi}{8}-i\sin\frac{3\pi}{8}}=\frac{2\cos^2\frac{3\pi}{16}+i2\sin\frac{3\pi}{16}\cos\frac{3\pi}{16}}{2\cos^2\frac{3\pi}{16}-i.2\sin\frac{3\pi}{16}.\cos\frac{3\pi}{16}}$$

$$=\frac{2\cos\frac{3\pi}{16}\left[\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}\right]}{2\cos\frac{3\pi}{16}\left[\cos\frac{3\pi}{16}-i\sin\frac{3\pi}{16}\right]} \qquad \because 1+\cos\theta=2\cos^2\frac{\theta}{2},\quad \sin\theta=2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$\left[\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right]^{\frac{8}{3}}=\left[\frac{\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}}{\cos\frac{3\pi}{16}-i\sin\frac{3\pi}{16}}\right]^{\frac{8}{3}}$$

$$\Rightarrow \frac{\cos\frac{8}{3}.\frac{3\pi}{16}+i\sin\frac{8}{3}.\frac{3\pi}{16}}{\cos\frac{8}{3}.\frac{3\pi}{16}-i\sin\frac{8}{3}.\frac{3\pi}{16}}=\frac{\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}}{\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}}=\frac{0+i(1)}{0-i(1)}=\frac{i}{-i}=-1$$

9. **Find all the roots of the equation** $x^{11}-x^7+x^4-1=0$

Sol. $x^{11}-x^7+x^4-1=0$

$x^7\left(x^4-1\right)+1\left(x^4-1\right)=0$

$\left(x^7+1\right)\left(x^4-1\right)=0$

$x^7+1=0 \Rightarrow x^7=-1=\cos\pi+i\sin\pi \qquad \because \text{cis}\pi=-1$

$\Rightarrow x=(-1)^{\frac{1}{7}}=\left(\cos\pi+i\sin\pi\right)^{\frac{1}{7}}$

$\Rightarrow x=\cos\frac{(2k\pi+\pi)}{7}+i\sin\left(\frac{2k\pi+\pi}{7}\right)$ where k=0,1,2,3,4,5,6

If k=0, $x=\cos\frac{\pi}{7}+i\sin\frac{\pi}{7}=cis\frac{\pi}{7}$

If k=1, $x=\cos\frac{3\pi}{7}+i\sin\frac{3\pi}{7}=cis\frac{3\pi}{7}$

If k=2, $x=\cos\frac{5\pi}{7}+i\sin\frac{5\pi}{7}=cis\frac{5\pi}{7}$

If k=3, x= $\cos\frac{7\pi}{7}+i\sin\frac{7\pi}{7}=cis\pi=-1$

If k=4, $x=\cos\frac{9\pi}{7}+i\sin\frac{9\pi}{7}=cis\frac{9\pi}{7}$

If k=5, $x=\cos\frac{11\pi}{7}+i\sin\frac{11\pi}{7}=cis\frac{11\pi}{7}$

If k=6, $x=\cos\frac{13\pi}{7}+i\sin\frac{13\pi}{7}=cis\frac{13\pi}{7}$

$x^4-1=0\Rightarrow x^4=1\Rightarrow x=(1)^{\frac{1}{4}}=\left(\cos0^0+i\sin0^0\right)^{\frac{1}{4}}$

$\Rightarrow x=\cos\frac{(2k\pi+0^0)}{4}+i\sin\left(\frac{2k\pi+0^0}{4}\right)$ where k=0,1,2,3

If k=0, $x=\cos0^0+i\sin0^0=1+1^0(0)=1$

If k=1, $x=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}=0+i(1)=i$

If k=2, $x=\cos\pi+i\sin\pi=-1+i(0)=-1$

If k=3, $x=\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}=0+i(-1)=-i$

$\therefore$ solution set= $\left\{1,-1,i,-i,cis\frac{\pi}{7},cis\frac{3\pi}{7},cis\frac{5\pi}{7},cis\frac{9\pi}{7},cis\frac{11\pi}{7},cis\frac{13\pi}{7}\right\}$

10. **Solve $x^9-x^5+x^4-1=0$**

Sol. $x^9-x^5+x^4-1=0$

$\Rightarrow x^5\left(x^4-1\right)+1\left(x^4-1\right)=0$

$\Rightarrow \left(x^4-1\right)\left(x^5+1\right)=0$

$x^4-1=0\Rightarrow x^4=1\Rightarrow x=(1)^{\frac{1}{4}}=\left(\cos0^0+i\sin0^0\right)^{\frac{1}{4}}$

$x=\cos\frac{(2k\pi+0^0)}{4}+i\sin\left(\frac{2k\pi+0^0}{4}\right)$ where k=0,1,2,3

If k=0, $x=\cos0^0+i\sin0^0+1+i(0)=1$

If k=1, $x=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}=0+i(1)=i$

If k=2, $x=\cos\pi+i\sin\pi=-1+i(0)=-1$

If k=3, $x=\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}=0+i(-1)=-i$

$x^5+1=0 \Rightarrow x^5=-1 \Rightarrow x=(-1)^{\frac{1}{5}}$

$\Rightarrow x=(\cos\pi+i\sin\pi)^{\frac{1}{5}}=\cos\left(\frac{2k\pi+\pi}{5}\right)+i\sin\left(\frac{2k\pi+\pi}{5}\right)$ where k = 0,1,2,3,4

If k=0, $x=\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}=cis\frac{\pi}{5}$

If k=1, $x=\cos\frac{3\pi}{5}+i\sin\frac{3\pi}{5}=cis\frac{3\pi}{5}$

If k=2, $x=\cos\frac{5\pi}{5}+i\sin\frac{5\pi}{5}=\cos\pi+i\sin\pi=-1$

If k=3, $x=\cos\frac{7\pi}{5}+i\sin\frac{7\pi}{5}=cis\frac{7\pi}{5}$

If k=4, $x=\cos\frac{9\pi}{5}+i\sin\frac{9\pi}{5}=cis\frac{9\pi}{5}$

$\therefore$ solution set$=\left\{1,-1,i,-i,cis\frac{\pi}{5},cis\frac{3\pi}{5},cis\frac{7\pi}{5},cis\frac{9\pi}{5}\right\}$

11. **If ‘n’ is an integer and** $z=cis\theta$ **then show that** $\frac{z^{2n}-1}{z^{2n}+1}=i\tan n\theta$

Sol. $\frac{z^{2n}-1}{z^{2n}+1}=\frac{(\cos\theta+i\sin\theta)^{2n}-1}{(\cos\theta+i\sin\theta)^{2n}+1}=\frac{\cos2n\theta+i\sin2n\theta-1}{\cos2n\theta+i\sin2n\theta+1}$

$=\frac{-(1-\cos2n\theta)+i\sin2n\theta}{1+\cos2n\theta+i\sin2n\theta}$

$=\frac{-(2\sin^2n\theta)+i(2\sin n\theta\cos n\theta)}{2\cos^2n\theta+i(2\sin n\theta\cos n\theta)}$

$\because 1-\cos2\theta=2\sin^2\theta$

$1+\cos2\theta=2\cos^2\theta$

$\sin2\theta=2\sin\theta\cos\theta$

$=\frac{2i^2\sin^2n\theta+i(2\sin n\theta\cos n\theta)}{2\cos^2n\theta+i(2\sin n\theta\cos n\theta)}$

$=\frac{2i\sin n\theta[\cos n\theta+i\sin n\theta]}{2\cos n\theta[\cos n\theta+i\sin n\theta]}$

$=i\frac{\sin n\theta}{\cos n\theta}$

$=i\tan n\theta$

12. **If** $(1+x)^n=a_0+a_1x+a_2x^2+\ldots\ldots+a_nx^n$ **then show that (i)** $a_0-a_2+a_4-a_6+\ldots\ldots=2^{\frac{n}{2}}\cos\frac{n\pi}{4}$

(ii) a_1-a_3+a_5…………….= $2^{\frac{n}{2}}\sin\frac{n\pi}{4}$

Sol. $(1+x)n=a_0+a_1x+a_2x^2+.....a_nx^n$

Let x=i

$\Rightarrow(1+i)^n=a_0+a_1(i)+a_2(i^2)+a_3(i^3)+a_4(i^4)+......a_n(i^n)$

$\Rightarrow a_0+a_1i+a_2i^2+a_3i^3+a_4i^4+a_5i^5+.....a_ni^n=(1+i)^n$

$\Rightarrow a_0+a_1i+a_2(-1)+a_3(-i)+a_4(1)+a_5(i)+......\left[\sqrt{2}\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)\right]^n$

$\Rightarrow a_0+a_1i-a_2-a_3i+a_4+a_5i-.........=\left(\sqrt{2}\right)^n\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^n$

$\Rightarrow(a_0-a_2+a_4.......)+i(a_1-a_3+a_5........)=2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4}+i\sin\frac{n\pi}{4}\right)$

$\Rightarrow(a_0-a_2+a_4......)+i(a_1-a_3+a_5-.........)=2^{\frac{n}{2}}\cos\frac{n\pi}{4}+2^{\frac{n}{2}}i\sin\frac{n\pi}{4}$

Equating real and imaginary parts

$a_0-a_2+a_4.........=2^{\frac{n}{2}}\cos\frac{n\pi}{4}$

$a_1-a_3+a_5.........=2^{\frac{n}{2}}\sin\frac{n\pi}{4}$

13. **If m,n are integers and** $x=\cos\alpha+i\sin\alpha,\ y=\cos\beta+i\sin\beta$ **then prove that**

$x^my^n+\frac{1}{x^my^n}=\cos(m\alpha+n\beta)\ \&\ x^my^n-\frac{1}{x^my^n}=2i\sin(m\alpha+n\beta)$

Sol. $x^m=(\cos\alpha+i\sin\alpha)^m=\cos m\alpha+i\sin m\alpha$

$y^n=(\cos\beta+i\sin\beta)^n=\cos n\beta+i\sin n\beta$

$\therefore x^my^n=(\cos m\alpha+i\sin m\alpha)(\cos m\beta+i\sin m\beta)$

$=\cos(m\alpha+n\beta)+i\sin(m\alpha+n\beta)$ ………….(1) $\because \text{cis}\theta,\text{cis}\phi=\text{cis}(\theta+\phi)$

$\frac{1}{x^my^n}=\cos(m\alpha+n\beta)-i\sin(m\alpha+n\beta)$ ………….. (2)

By adding (1) & (2)

$x^my^n+\frac{1}{x^my^n}=\cos(m\alpha+n\beta)+i\sin(m\alpha+n\beta)+\cos(m\alpha+n\beta)-i\sin(m\alpha+n\beta)$

$=2\cos(m\alpha+n\beta)$

By subtracting (1) & (2)

$x^my^n-\frac{1}{x^my^n}=\cos(m\alpha+n\beta)+i\sin(m\alpha+n\beta)-\cos(m\alpha+n\beta)+i\sin(m\alpha+n\beta)$

$=2i\sin(m\alpha+n\beta)$

$\therefore x^my^n+\frac{1}{x^my^n}=2\cos(m\alpha+n\beta)$

$x^my^n-\frac{1}{x^my^n}=2i\sin(m\alpha+n\beta)$

14. **If 1,w,w^2 are the cube roots of unity prove that**

(i) $\left(1-w^2+w^2\right)^6+\left(1-w^2+w\right)^6=128=\left(1-w+w^2\right)^7+\left(1-w^2+w\right)^7$

(ii) $(a+b)\left(aw+bw^2\right)\left(aw^2+bw\right)=a^3+b^3$

(iii) $x^2+4x+7=0$ **when** $x=w-w^2-2$

Sol. (i) $\left(1-w+w^2\right)^6+\left(1-w^2+w\right)^6=(-w-w)^6+\left(-w^2-w^2\right)^6$

$=(-2w)^6+\left(-2w^2\right)^6$

$=2^6\left(w^6+w^{12}\right)$

$=2^6\left(\left(w^3\right)^2+\left(w^3\right)^4\right)$

$=2^6(1+1)=2^6.2$

=64x2=128

$\left(1-w+w^2\right)^7+\left(1+w-w^2\right)^7=(-w-w)^7+\left(-w^2-w^2\right)^7=(-2w)^7+\left(-2w^2\right)^7$

$=(-2)^7\left(w^7+w^{14}\right)=(-2)^7\left(w+w^2\right)$

$=-128(-1)=128$

(ii) $(a+b)\left(aw+bw^2\right)\left(aw^2+bw\right)=(a+b)\left(a^2w^3+abw^4+abw^2+b^2w^3\right)$

$\Rightarrow(a+b)\left(a^2(1)+abw+abw^2+b^2\right)=(a+b)\left(a^2+ab\left(w+w^2\right)+b^2\right)$

$=(a+b)\left(a^2+ab(-1)+b^2\right)$

$=(a+b)\left(a^2-ab+b^2\right)=a^3+b^3$

(iii) $x=w-w^2-2\Rightarrow x+2=w-w^2$

$\Rightarrow(x+2)^2=\left(w-w^2\right)^2 \quad \Rightarrow x^2+4x+4=w^2+w^4-2w^3$

$\Rightarrow x^2+4x+4=w^2+w-2$

$=-1-2=-3$

$\Rightarrow x^2+4x+4=-3$

$\Rightarrow x^2+4x+7=0$

CHAPTER - 3

QUADRATIC EXPRESSIONS

WEIGHTAGE : (2 + 4 MARKS)

VERY SHORT QUESTIONS (2 MARKS)

1. Find the quadratic equation whose roots are 7+ 2$\sqrt{5}$ and 7 - 2$\sqrt{5}$.

Solu: Let $\alpha = 7 + 2\sqrt{5}$ and β = 7 - 2$\sqrt{5}$.

Then $\alpha + \beta = 7 + 2\sqrt{5} + 7 - 2\sqrt{5}$ = 14

$\alpha\beta$ = $(7 + 2\sqrt{5})(7 - 2\sqrt{5})$ = 49 - 20 = 29

If α , β are roots then $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ is the quadratic equation.

The required quadratic equation is **$x^2 - 14x + 29 = 0$.**

2. Find the quadratic equation whose roots are -3 $\pm$ 5i.

Solu: Let $\alpha = -3 + 5i$ and β = - 3 – 5i

Then $\alpha + \beta$ = $(-3 + 5i) + (-3 - 5i)$ = - 6

$\alpha\beta = (-3 + 5i)(-3 - 5i) = 9 + 25 = 34$

If α , β are roots then $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ is the quadratic equation.

The required quadratic equation is **$x^2 + 6x + 34 = 0$.**

3. Find the quadratic equation whose roots are $\frac{p-q}{p+q}$, $\frac{-(p+q)}{p-q}$ (p $\neq \pm q$).

Solu : Let $\alpha = \frac{p-q}{p+q}$ and $\beta = \frac{-(p+q)}{p-q}$

Then $\alpha + \beta = \frac{(p-q)}{p+q} + \frac{-(p+q)}{p-q} = \frac{(p-q)^2-(p+q)^2}{p^2-q^2} = \frac{-4pq}{p^2-q^2}$

$\alpha\beta = (\frac{p-q}{p+q})(\frac{-(p+q)}{p-q}) = -1$

If α, β are roots then $x^2 - (\alpha+\beta)x + \alpha\beta = 0$ is the quadratic equation.

The required quadratic equation is $\mathbf{x^2 - (\frac{-4pq}{p^2-q^2)})\, x - 1 = 0.}$

$\mathbf{(p^2 - q^2)\, x^2 + 4pqx - (p^2 - q^2) = 0}$

4. If α, β are roots of the equation $ax^2 + bx + c = 0$ then find the values of

i) $\frac{1}{\alpha} + \frac{1}{\beta}$ ii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ iii) $\alpha^2 + \beta^2$

Solu :If α, β are roots of the equation $ax^2 + bx + c = 0$ then

$\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

I) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{-b}{c}$

II) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}} = \frac{b^2-2ac}{c^2}$

III) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2-2ac}{a^2}$

5. For what values of m the equation $(m+1)x^2 + 2(m+3)x + m + 8 = 0$ has equal roots.

Solu :

Roots are equal $\Rightarrow \Delta = 0 \Rightarrow b^2 - 4ac = 0$.

Here a = m+1 , b= 2m+6, c = m+8

$\Rightarrow (2m+6)^2 - 4(m+1)(m+8) = 0 \Rightarrow 4m^2 + 24m + 36 - 4(m^2 + 9m + 8) = 0$

$\Rightarrow 4m^2 + 24m + 36 - 4m^2 - 36m - 32 = 0 \Rightarrow -12m + 4 = 0 \Rightarrow \mathbf{m = \frac{1}{3}}$

6. If the equation $x^2 - 15 - m(2x-8) = 0$ has equal roots find value of m.

Solu :

Roots are equal $\Rightarrow \Delta = 0 \Rightarrow b^2 - 4ac = 0$.

Given equation can be rewritten as $x^2 - 2mx + 8m - 15 = 0$

Here $a = 1$, $b = -2m$, $c = 8m - 15$

$\Longrightarrow (-2m)^2 - 4(1)(8m - 15) = 0 \Longrightarrow 4m^2 - 32m + 60 = 0$

$\Longrightarrow m^2 - 8m + 15 = 0 \Longrightarrow m^2 - 5m - 3m + 15 = 0$

$\Longrightarrow (m-5)(m-3) = 0 \Longrightarrow$ **$m = 5$, $m = 3$**

7. At what value of x the expression $2x - 7 - 5x^2$ has maximum and also find the maximum value .

Solu: Given expression $2x - 7 - 5x^2$

Here $a = -5$, $b = 2$, $c = -7$

Since $a < 0$,The expression has absolute maximum at **x** = $\frac{-b}{2a} = \frac{-2}{2(-5)} = \frac{1}{5}$

Maximum value = $\frac{4ac - b^2}{4a} = \frac{4(-5)(-7) - (2)^2}{4(-5)} = \frac{-34}{5}$.

8. Find the maximum or minimum of the expression $x^2 - x + 7$

Solu: Given expression $x^2 - x + 7$

Here $a = 1$, $b = -1$, $c = 7$

Since $a > 0$,The expression has absolute minimum at **x** = $\frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$

Minimum value = $\frac{4ac - b^2}{4a} = \frac{4(1)(7) - (-1)^2}{4(1)} = \frac{27}{4}$.

9. Find the changes in the sign of expression $x^2 - 5x + 6$.

Solu: Case - I) $x^2 - 5x + 6 > 0 \Longrightarrow x^2 - 3x - 2x + 6 > 0$

$\Longrightarrow x(x-3) - 2(x-3) > 0 \Longrightarrow (x-2)(x-3) > 0$

$(x - \alpha)(x - \beta) > 0 \Rightarrow x < \alpha$ or $x > \beta$

$x < 2$ or $x > 3$

Case - Ii) $x^2 - 5x + 6 < 0 \Longrightarrow x^2 - 3x - 2x + 6 < 0$

$\Longrightarrow x(x-3) - 2(x-3) < 0 \Longrightarrow (x-2)(x-3) < 0$

$(x - \alpha)(x - \beta)$, $0 \Rightarrow \alpha < x < \beta$

$2 < x < 3$

Hence for **$x < 2$ or $x > 3$** the expression **is positive** and for

$2 < x < 3$ the expression is **negative.**

10. For what values of x the expression $15 + 4x - 3x^2$ is negative.

Solu: $15 + 4x - 3x^2 < 0 \Rightarrow -(3x^2 - 4x - 15) < 0$

$\Rightarrow 3x^2 - 4x - 15 > 0 \Rightarrow 3x^2 - 9x + 5x - 15 > 0$

$\Rightarrow 3x(x-3) + 5(x-3) > 0 \Rightarrow (3x+5)(x-3) > 0$

$(x - \alpha)(x - \beta) > 0 \Rightarrow x < \alpha$ or $x > \beta$

$x < \frac{-5}{3}$ or $x > 3$

11. Find a quadratic equation ,the sum of whose roots is 1 and sum of squares of the roots is 13.

Solu: Let α , β be the roots of the equation .

Given $\alpha + \beta = 1$ and $\alpha^2 + \beta^2 = 13$

$\boldsymbol{\alpha^2} + \boldsymbol{\beta^2} = (\alpha + \beta)^2 - 2\alpha\beta \Rightarrow (1)^2 - 2(\alpha\beta) = 13$

$2\alpha\beta = -12 \Rightarrow \boldsymbol{\alpha\beta = -6}$

If α , β are roots then $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ is the quadratic equation.

The required equation is $x^2 - (1)x + (-6) = 0 \Rightarrow \mathbf{x^2 - x - 6 = 0}$

12. If $x^2 - 6x + 5 = 0$ and $x^2 - 12x + p = 0$ have a common root then find p.

Solu: Given equation is $x^2 - 6x + 5 = 0 \Rightarrow x^2 - 5x - x + 6 = 0$

$\Rightarrow x(x-5) - 1(x-5) = 0 \Rightarrow (x-5)(x-1) = 0$

$\Rightarrow x = 1$ or $x = 5$.

If $x = 1$ is a common root then $x^2 - 12x + p = 0 \Rightarrow 1 - 12 + p = 0 \Rightarrow \mathbf{p = 11}$

If $x = 5$ is a common root then $x^2 - 12x + p = 0 \Rightarrow 25 - 60 + p = 0 \Rightarrow \mathbf{p = 35.}$

13. If $x^2 + bx + c = 0$ and $x^2 + cx + b = 0$ have a common root then show that $b + c + 1 = 0$

Solu : Let α be common root of both the equations. Then

$\alpha^2 + b\alpha + c = 0$ --- (1) and $\alpha^2 + c\alpha + b = 0$ ------(2)

Solving these two equations

(1) - (2) $\Rightarrow \alpha^2 + b\alpha + c - \alpha^2 - c\alpha - b = 0$

$\Rightarrow b\alpha - c\alpha + c - b = 0$

$\Rightarrow \alpha(b - c) - (b - c) = 0$

$\Rightarrow \alpha = \frac{b-c}{b-c} = 1$

Substitute $\alpha = 1$ in eqn ---(1)

$\Rightarrow 1 + b = c = 0$

14 Prove that roots of (x-a) (x- b) = h^2 are always real.

Solu :Given equation is (x-a) (x-b) = h^2

$\Rightarrow x^2 - (a+b)x + ab = h^2$

$\Rightarrow x^2 - (a+b)x = h^2 - ab$

Here a = 1 b = -(a+b) c = ab - h^2

Discriminant $\Delta = b^2 - 4ac = (a+b)^2 - 4(1)(ab - h^2) = (a+b)^2 - 4ab + 4h^2$

$= (a - b)^2 + 4h^2 \geq 0$

Roots are always real.

Model questions :

1. **Find the value of m for which the following equations have equal roots.**

   I) $X^2 + (m+3)x + (m+6) = 0$ ii) $(3m+1)x^2 + 2(m+1)x + m = 0$

   Iii) $(2m+1)x^2 + 2(m+3)x + (m+5) = 0$

2. **Find the maximum or minimum of the following expressions**

   I) $3x^2 + 4x + 1 = 0$ ii) $4x - x^2 - 10 = 0$ iii) $x^2 + 5x + 6 = 0$

3. **Determine the sign of expressions**

   I) $X^2 - 5x + 14$ ii) $3x^2 + 4x + 4$.

4. **Find a quadratic equation ,the sum of whose roots is 7 and sum of squares of the roots is 25.**

SHORT ANSWER QUESTIONS (4 MARKS)

1. **Prove that $\frac{1}{3x+1} + \frac{1}{x+1} - \frac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4 if**

x is real.

Solu: Let y = $\frac{1}{3x+1} + \frac{1}{x+1} - \frac{1}{(3x+1)(x+1)}$ = $\frac{x+1+3x+1-1}{(3x+1)(x+1)}$

$\Rightarrow$Y = $\frac{4x+1}{(3x+1)\,(x+1)}$

$\Rightarrow$ y(3x+1) (x+1) = 4x+1

$\Rightarrow$ y($3x^2$ +3x +x + 1) = 4x + 1

$\Rightarrow$ $3x^2$ y +4xy + y = 4x + 1

$\Rightarrow$ $3x^2$ y +4xy + y - 4x - 1 = 0

$\Rightarrow$ $3x^2$ y + (4y -4)x +(y - 1) = 0

It is in the form of $ax^2 + bx + c = 0$ where a = 3y , b = 4y - 4 , c = y - 1

Since x is real $\Rightarrow \Delta \geq$ **0** $\Rightarrow$ b^2 - 4ac $\geq$ **0.**

$\Rightarrow$(4y -4)^2 - 4(3y) (y - 1) $\geq$ **0.**

$\Rightarrow$ $16y^2$ - 32 y + 16 -$12y^2$ + 12y $\geq$ **0.**

$\Rightarrow$ $4y^2$ - 20y + 16 $\geq$ **0.**$\Rightarrow$ 4(y^2 - 5y+ 4) $\geq$ **0.**

$\Rightarrow$ Y^2 - 5y + 4 $\geq$ **0.**$\Rightarrow$ y^2 - 4y -y + 4 $\geq$ **0.**

$\Rightarrow$ Y(y -4) - 1(y - 4) $\geq$ **0.**$\Rightarrow$ (y-4) (y-1) $\geq$ **0.**

Y does not lie between 1 and 4 .

(x -α) (x - β)≥ 0 ⇒ x≤ α or x≥ β

Hence the given expression does not lie between 1 and 4 .

2. If x is real prove that $\frac{x}{x^2-5x+9}$ lies between $\frac{-1}{11}$ and 1

Solu : Y = $\frac{x}{x^2-5x+9}$

$\Rightarrow$ y($x^2 - 5x + 9$) = x $\Rightarrow$ yx^2 - 5xy + 9y = x

$\Rightarrow$ yx^2 - 5xy + 9y - x = 0

$\Rightarrow$ yx^2 + (-5y - 1) x + 9y = 0

It is in the form of $ax^2 + bx + c = 0$ where a = y , b = -5y - 1 , c = 9y

Since x is real $\Rightarrow \Delta \geq$ **0** $\Rightarrow$ b^2 - 4ac $\geq$ **0.**

$\Rightarrow$(-5y - 1)^2 - 4(y) (9y) $\geq$ **0.** $\Rightarrow$ $25y^2$ + 10 y + 1 - $36y^2 \geq$ **0.**

$\Rightarrow -11y^2 + 10y + 1 \quad \geq \mathbf{0}. \Rightarrow \quad -(11y^2 - 10y - 1) \geq \mathbf{0}.$

$\Rightarrow 11Y^2 - 10y - 1 \quad \leq \mathbf{0}. \Rightarrow \quad 11y^2 - 11y + y - 1 \leq \mathbf{0}.$

$\Rightarrow 11\,Y(y-1) + 1(y-1) \quad \leq \mathbf{0}. \Rightarrow \quad (11y+1)(y-1) \leq \mathbf{0}.$

Y lies between $\frac{-1}{11}$ and 1

$(\mathbf{x} - \alpha)(\mathbf{x} - \beta) < \mathbf{0} \Rightarrow \alpha < \mathbf{x} < \beta$

Hence the given expression lies between $\frac{-1}{11}$ and 1

3. **Show that none of the values of the function $\frac{x^2 + 34x - 71}{x^2 + 2x - 7}$ over R lies between 5 and 9**

Solu: Let y = $\frac{x^2 + 34x - 71}{x^2 + 2x - 7} \Rightarrow y(x^2 + 2x - 7) = x^2 + 34x - 71$

$\Rightarrow y\,x^2 + 2x\,y - 7y = x^2 + 34x - 71$

$\Rightarrow y\,x^2 + 2x\,y - 7y - x^2 - 34x + 71 = 0$

$\Rightarrow (y-1)\,x^2 + (2y - 34)x + 71 - 7y = 0$

It is in the form of $ax^2 + bx + c = 0$ where a = y - 1 , b = 2y - 34 , c = -7 y +71

Since x is real $\Rightarrow \Delta \geq \mathbf{0} \Rightarrow b^2 - 4ac \geq \mathbf{0}.$

$\Rightarrow (2y - 34)^2 - 4(71 - 7y)(y - 1) \geq \mathbf{0}.$

$\Rightarrow 4y^2 - 136y + 1156 - 4(71y - 7y^2 - 71 + 7y) \geq \mathbf{0}.$

$\Rightarrow 4y^2 - 136y + 1156 - 312y + 28y^2 + 284 \geq \mathbf{0}.$

$\Rightarrow 32y^2 - 448y + 1440 \quad \geq \mathbf{0}. \Rightarrow \quad 32(y^2 - 14y + 45) \geq \mathbf{0}.$

$\Rightarrow Y^2 - 14y + 45 \quad \geq \mathbf{0}. \Rightarrow \quad y^2 - 9y - 5y + 45 \geq \mathbf{0}.$

$\Rightarrow Y(y-9) - 5(y-9) \quad \geq \mathbf{0}. \Rightarrow \quad (y-5)(y-9) \geq \mathbf{0}.$

Y does not lie between 5 and 9.

$(\mathbf{x} - \alpha)(\mathbf{x} - \beta) \geq \mathbf{0} \Rightarrow \mathbf{x} \leq \alpha$ **or** $\mathbf{x} \geq \beta$

Hence the given expression does not lie between 5 and 9.

4. **If the expression $\frac{x - p}{x^2 - 3x + 2}$ takes all real values for $x \in \mathbf{R}$ then**

find the bounds of p

Solu : Let Y = $\frac{x-p}{x^2-3x+2}$

$\Rightarrow$ y($x^2 - 3x + 2$) = x - p $\Rightarrow$ yx^2 - 3xy + 2y = x - p

$\Rightarrow$ yx^2 - 3xy + 2y - x + p = 0

$\Rightarrow$ yx^2 - (3y +1) x + 2y + p = 0

It is in the form of $ax^2 + bx + c = 0$ where a = y , b = -3y - 1 , c = 2y + p

Since x is real $\Rightarrow \Delta \geq \mathbf{0}$ $\Rightarrow$ $b^2 - 4ac \geq \mathbf{0}$.

$\Rightarrow (-5y - 1)^2 - 4(y)(2y + p) \geq \mathbf{0}$. $\Rightarrow$ $9y^2 + 6y + 1 - 8y^2 - 4py \geq \mathbf{0}$.

$\Rightarrow$ $y^2 + 6y + 1 - 4py \geq \mathbf{0}$. $\Rightarrow$ $y^2 + (6 - 4p)y + 1 \geq \mathbf{0}$.

$\forall$ x $\in$R ,sign of the expression is > 0 , coeffecient of x^2 is > 0 $\Rightarrow \Delta < \mathbf{0}$

Since sign of the expression is > 0 , coeffecient of y^2 is > 0 $\Rightarrow \Delta < 0$

$\Rightarrow b^2 - 4ac < 0$. here a = 1 , b = 6-4p , c = 1

$\Rightarrow$ $(6 - 4p)^2 - 4(1)(1) < 0$ $\Rightarrow$ $16p^2 - 48p + 36 - 4 < 0$

$\Rightarrow$ $16p^2 - 48p + 32 < 0$ $\Rightarrow$ $16(p^2 - 3p + 2) < 0$.

$\Rightarrow$ $p^2 - 2p - p + 2 < 0$. $\Rightarrow$ $(p - 1)(p - 2) < 0$.

$\Rightarrow \mathbf{1} < \mathbf{p} < 2$

$(x - \alpha)(x - \beta) < 0 \Rightarrow \alpha < x < \beta$

Hence p $\in$ $(\mathbf{1}, \mathbf{2})$

5. Find the maximum and minimum value of $\frac{x^2 + 14x + 9}{x^2 + 2x + 3}$

Solu : Let y = $\frac{x^2 + 14x + 9}{x^2 + 2x + 3}$ $\Rightarrow$ y($x^2 + 2x + 3$) = $x^2 + 14x + 9$

$\Rightarrow$y x^2 + 2x y+3y = x^2 + 14x +9

$\Rightarrow$y x^2 + 2x y+3y - x^2 - 14x - 9 = 0

$\Rightarrow$ (y-1) x^2 + (2y - 14)x + (3y - 9) = 0

It is in the form of $ax^2 + bx + c = 0$ where a = y - 1 , b = 2y - 14 , c = 3 y - 9

Since x is real $\Rightarrow \Delta \geq \mathbf{0}$ $\Rightarrow$ $b^2 - 4ac \geq \mathbf{0}$.

$\Rightarrow (2y - 14)^2 - 4(3y - 9)(y - 1) \geq$ **0.**

$\Rightarrow 4y^2 - 56y + 196 - 4(3y^2 - 3y + 9 - 9y) \geq$ **0.**

$\Rightarrow 4y^2 - 56y + 196 - 12y^2 + 36y + 12y - 36 \geq$ **0.**

$\Rightarrow -8y^2 - 8y + 160 \geq$ **0.** $\Rightarrow -8(y^2 + y - 20) \geq$ **0.**

$\Rightarrow Y^2 + y - 20 \leq$ **0.** $\Rightarrow y^2 + 5y - 4y - 20 \leq$ **0.**

$\Rightarrow Y(y + 5) - 4(y + 5) \leq$ **0.** $\Rightarrow (y + 5)(y - 4) \leq$ **0.**

Y does not lie between -5 and 4

$(x - \alpha)(x - \beta) \leq 0 \Rightarrow \alpha \leq x \leq \beta$

Maximum value is 4 and minimum value is -5

6. If the equations $x^2 + ax + b = 0$ and $x^2 + cx + d = 0$ have a common root and the first

equation has equal roots then show that 2(b+d) =ac

Solu: Given equations $x^2 + ax + b = 0$ ------(1) $x^2 + cx + d = 0$ ------(2)

Let α be common root of (1) and (2)

$\alpha^2 + a\alpha + b = 0$ -------(3) $\alpha^2 + c\alpha + d = 0$ ---------(4)

Since Eqn (1) has equal roots let them be α , α

Sum of the roots $\alpha + \alpha = -a \Rightarrow 2\alpha = -a \Rightarrow \alpha = -a/2$

Product of roots $\alpha\alpha = b \Rightarrow \alpha^2 = b$

Substitute α and α^2 in (4) $\Rightarrow b + c(-a/2) + d = 0$

$\Rightarrow \frac{2b - ac + 2d}{2} = 0 \Rightarrow 2b - ac + 2d = 0$

$\Rightarrow$ **2(b+d) = ac.**

7. If $c \neq ab$ and the roots of $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are equal then show

that $a^3 + b^3 + c^3 = 3abc$ or a = 0.

Solu : Given equation is $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$

It is in the form of $ax^2 + bx + c = 0$ where $a = c^2 - ab$, $b = -2(a^2 - bc)$, $c = b^2 - ac$

Given that roots are equal $\Rightarrow \Delta = \mathbf{0} \Rightarrow b^2 - 4ac = \mathbf{0}.$

$\Longrightarrow$ $[-2(a^2 - bc)]^2 - 4(c^2 - ab)(b^2 - ac) = 0$

$\Longrightarrow$ $4(a^4 + b^2c^2 - 2a^2bc) - 4(c^2b^2 - ac^3 - ab^3 + a^2bc) = 0$

$\Longrightarrow$ $4[a^4 + b^2c^2 - 2a^2bc - c^2b^2 + ac^3 + ab^3 - a^2bc] = 0$

$\Longrightarrow$ $4a(a^3 - 3abc + c^3 + b^3) = 0$

$\Longrightarrow$ **$a = 0$ or $a^3 + b^3 + c^3 = 3abc$**

8. Solve $\sqrt{\frac{x}{x-3}} + \sqrt{\frac{x-3}{x}} = \frac{5}{2}$

Solu : Let $\sqrt{\frac{x}{x-3}} = t$ $\sqrt{\frac{x-3}{x}} = \frac{1}{t}$

Given $t + \frac{1}{t} = \frac{5}{2} \Longrightarrow \frac{t^2+1}{t} = \frac{5}{2}$

$\Longrightarrow 2t^2 + 2 = 5t \Longrightarrow 2t^2 - 5t + 2 = 0 \Longrightarrow 2t^2 - 4t - t + 2 = 0$

$\Longrightarrow 2t(t-2) - (t-2) = 0 \Longrightarrow (2t-1)(t-2) = 0$

$\Longrightarrow t = 1/2$, $t = 2$

$t = 1/2 \Longrightarrow \sqrt{\frac{x}{x-3}} = 1/2 \Longrightarrow \frac{x}{x-3} = \frac{1}{4}$

$\Longrightarrow 4x = x - 3 \Longrightarrow 3x = -3 \Longrightarrow$ **$x = -1$**

$t = 2 \Longrightarrow \sqrt{\frac{x}{x-3}} = 2 \Longrightarrow \frac{x}{x-3} = 4$

$\Longrightarrow x = 4x - 12 \Longrightarrow 3x = 12 \Longrightarrow$ **$x = 4$**

9. Let a , b , c $\in$R and **a** $\neq$ **0 such that the equation $ax^2 + bx + c = 0$ has real roots** α, β and $\alpha < \beta$ then

I) For $\alpha < x < \beta$, $ax^2 + bx + c$ and 'a' have opposite signs.

ii) For $x < \alpha$ and $x > \beta$, $ax^2 + bx + c$ and 'a' have same signs.

Solu: Let α, β are the roots of $ax^2 + bx + c = 0$ then $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

$ax^2 + bx + c = a(x^2 + \frac{b}{a}x + \frac{c}{a}) = a(x^2 - (-\frac{b}{a})x + \frac{c}{a}) = a(x^2 - (\alpha + \beta)x + \alpha\beta)$

$\Longrightarrow ax^2 + bx + c = a(x-\alpha)(x-\beta) \Longrightarrow \frac{ax^2 + bx + c}{a} = (x-\alpha)(x-\beta)$ -------(1)

I) Given $\alpha < x < \beta \Rightarrow \alpha < x$, $x < \beta \Rightarrow$ **x >**α , $x < \beta$

$\Rightarrow (x - \alpha) > 0$, $(x - \beta) < 0 \Rightarrow (x - \alpha)(x - \beta) < 0$

From eqn (1) $\frac{ax^2 + bx + c}{a} < 0$, **$ax^2 + bx + c$ and 'a' have opposite signs.**

Ii) Case - (a) **x <**α and we have $\alpha < \beta$

$\Rightarrow x < \beta \Rightarrow (x - \alpha) < 0$,$(x - \beta) < 0$

$\Rightarrow (x - \alpha)(x - \beta) > 0$

Case (b) $x > \beta$, β**>**αType equation here.

$\Rightarrow x > \beta$, $\beta > \alpha \Rightarrow x$ **>**α

$\Rightarrow x$ **>**α , $x > \beta \Rightarrow (x - \alpha) > 0$,$(x - \beta) > 0$

$\Rightarrow (x - \alpha)(x - \beta) > 0$

From eqn (1) $\frac{ax^2 + bx + c}{a} > 0$, **$ax^2 + bx + c$ and 'a' have same signs.**

10. **Let a , b , c** $\in$R and **a $\neq$ 0 then the roots of $ax^2 + bx + c = 0$ are nonreal complex numbers if and only if $ax^2 + bx + c$ and 'a' have same signs.**

Solu : Given quadratic equation is $ax^2 + bx + c + 0$

Suppose that the equation has nonreal complex roots then $b^2 - 4ac < 0$

Now $ax^2 + bx + c = a(x^2 + \frac{b}{a}x + \frac{c}{a}) = a(x^2 + 2(\frac{b}{2a})x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2} + \frac{c}{a})$

$\Rightarrow ax^2 + bx + c = a[(x + \frac{b}{2a})^2 - \frac{b^2}{4a^2} + \frac{c}{a}]$

$= a[(x + \frac{b}{2a})^2 + \frac{4ac - b^2}{4a^2}]$

$\Rightarrow \frac{ax^2 + bx + c}{a} = [(x + \frac{b}{2a})^2 + \frac{4ac - b^2}{4a^2}] > 0$ (since $b^2 - 4ac < 0$)

$\frac{ax^2 + bx + c}{a} > 0$ $\therefore$**$ax^2 + bx + c$ and 'a' have same signs.**

Conversely suppose that $ax^2 + bx + c$ and 'a' have same signs (i.e) $\frac{ax^2 + bx + c}{a} > 0$, $\forall x \in$R.

$\Rightarrow \frac{ax^2 + bx + c}{a} = [(x + \frac{b}{2a})^2 + \frac{4ac - b^2}{4a^2}] > 0 \quad \forall x \in$R.

On taking $x = -\frac{b}{2a}$ then $\frac{4ac - b^2}{4a^2} > 0 \Rightarrow 4ac - b^2 > 0$

$\Longrightarrow$ $b^2 - 4ac < 0$

Hence $ax^2 + bx + c = 0$ has nonreal complex roots.

Model Problems :

1. **Find the range of the expression** $\frac{x^2 + x + 1}{x^2 - x + 1}$ for $x \in R$
2. **Determine the range of the expression** $\frac{2x^2 - 6x + 5}{x^2 - 3x + 2}$ for $x \in R$
3. **Determine the range of the expression** $\frac{x + 2}{2x^2 + 3x + 6}$ for $x \in R$
4. **Determine the range of the expression** $\frac{(x+1)(x+2)}{x + 3}$ for $x \in R$
5. **Solve $2\left(x + \frac{1}{x}\right)^2 - 7\left(x + \frac{1}{x}\right) + 5 = 0$ when $x \neq 0$.**

****

CHAPTER – 4

THEORY OF EQUATIONS

WEIGHTAGE : (2 +7 = 9 MARKS.)

VERY SHORT ANSWER QUESTIONS (2 MARKS)

1. Find the polynomial equation whose roots are $2\pm\sqrt{3}$, $1 \pm 2i$.

Solu: Given roots are $2\pm\sqrt{3}$, $1 \pm 2i$.

If $\alpha, \beta, \gamma, \delta$, are roots then $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta) = 0$ is the equation

R

equired equation is

$[x-(2+\sqrt{3})]\,[x-(2+\sqrt{3})]\,\{x-(1+2i)]\,[x-(1-2i)] = 0$

$[(x-2)-\sqrt{3}]\,[(x-2)+\sqrt{3}]\,[(x-1)+2i[\,(x-1)-2i] = 0$

$[(x-2)^2-(\sqrt{3})^2\,]\,[(x-1)^2-(2i)^2\,] = 0$ (since $(a+b)(a-b) = a^2 - b^2$)

$(x^2-4x+1)(x^2-2x+5) = 0$ (since $i^2 = -1$)

$X^4 - 6x^3 + 14x^2 - 22x + 5 = 0$

2. Form the monic polynomial equation of degree 3 whose roots are 2 , 3 , 6.

Solu: The polynomial equation whose roots are 2 , 3 , 6 is

$\Rightarrow (x-2)(x-3)(x-6) = 0$

$\Rightarrow (x^2-5x+6)(x-6) = 0$

$\Rightarrow X^3 - 11x^2 + 36x - 36 = 0$

If α, β, γ are roots then $(x-\alpha)(x-\beta)(x-\gamma) = 0$ is the equation

3 If α, β, γ are roots of $4x^3 - 6x^2 + 7x + 3 = 0$ then find the value of $\alpha\beta + \beta\gamma + \gamma\alpha$

Solu : Given equation is $4x^3 - 6x^2$

$+7x + 3 = 0$

If α, β, γ are roots then the value of $\alpha\beta + \beta\gamma + \gamma\alpha = S_2 = P_2$

$\Rightarrow X^3 - \frac{6}{4}x^2 + \frac{7}{4}x + \frac{3}{4} = 0$

$\Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = S_2 = P_2 = \mathbf{\frac{7}{4}}$

4 If 1 , 1 , α are roots of $x^3 - 6x^2 + 9x - 4 = 0$ then find α.

Solu : Given 1 , 1 , α are roots of $x^3 - 6x^2 + 9x - 4 = 0$

Then S_1= sum of the roots = $-P_1 = -\left(\frac{a_1}{a_0}\right) = -(-6) = 6$

$\Rightarrow 1+1+\alpha = 6.$

$\Rightarrow \alpha = \mathbf{4}$

5.If -1 , 2 , α are the roots of the equation $2x^3 + x^2 - 7x - 6 = 0$ then find α.

Solu : Given - 1 , 2 , α are roots of $2x^3 + x^2 - 7x - 6 = 0$

Then S_1 = sum of the roots = $-P_1 = -\left(\frac{a_1}{a_0}\right) = -\frac{1}{2}$

$\Rightarrow -1+2+\alpha = -\frac{1}{2} \Rightarrow \boldsymbol{\alpha} = -\mathbf{\frac{3}{2}}$

6. If 1 , -2 , 3 are roots of the equation $x^3 - 2x^2 + ax + 6 = 0$ then find a

Solu : If α, β, γ are roots then $S_2 = \alpha\beta + \beta\gamma + \gamma\alpha = P_2$

$S_2 = a = (1)(-2) + (-2)(3) + (3)(1) = -2 - 6 + 3 = -5$

Hence **a = -5.**

7. **If the product of roots of $4x^3 + 16x^2 - 9x - a = 0$ is 9 then find a**.

Solu : Given $4x^3 + 16x^2 - 9x - a = 0$,

: If α, β, γ are roots then S_3= product of roots

Then S_3 = product of the roots = $-P_3 = -\left(\frac{a_3}{a_0}\right)$

$S_3 = \alpha\beta\gamma = 9 \Rightarrow -\left(-\frac{a}{4}\right) = 9 \Rightarrow$ **a = 36**.

8. **If α, β , 1 are roots of $x^3 - 2x^2 - 5x + 6 = 0$ then find α, β .**

Solu: Given α , β ,1 are roots of $x^3 - 2x^2 - 5x + 6 = 0$

Now $S_1 = 2$

S_1 = sum of the roots = - P_1 = $-\left(\frac{a_1}{a_0}\right)$

$\Rightarrow \alpha + \beta + 1 = 2$

S_3 = product of the roots = - P_3 = $-\left(\frac{a_3}{a_0}\right)$

$\Rightarrow \alpha + \beta = 1$ eqn(1)

Now $S_3 = -6$

$\Rightarrow \alpha . \beta .1 = -6$

$\Rightarrow (\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\ \alpha . \beta$

$\Rightarrow (\alpha - \beta)^2 = 1 - 4(-6) = 25$

$\Rightarrow (\alpha - \beta) = 5 \longrightarrow$ eqn 2

Solving eqns (1) and (2)

$\alpha + \beta = 1$

$\alpha - \beta = 5$ We get $\boldsymbol{\alpha}$ **= 3 and** $\boldsymbol{\beta}$ **= - 2.**

9. **If $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$ are roots of $x^3 - 2x^2 + 3x - 4 = 0$ then find the value of**

i) $\sum \alpha^2 \beta^2$ (ii) $\sum \alpha^2 \beta + \sum \alpha \beta^2$.

Solu: Given α , β , γ are roots of $x^3 - 2x^2 + 3x - 4 = 0$

$\mathbf{S_1} = \alpha + \beta + \gamma = 2$

$S_2 = \alpha\beta + \beta\gamma + \gamma\alpha = 3$

$S_3 = \alpha\beta\gamma = 4.$

i) $\sum \alpha^2 \beta^2 = \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$

$= (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2(\alpha\beta\gamma)(\alpha + \beta + \gamma)$

$= 3^2 - 2(4)(2) = 9 - 16 =$ **-7**

ii) $\sum \alpha^2 \beta + \sum \alpha \beta^2 = \mathbf{S_1} S_2 - 3 S_3$

= 2(3) – 3(4) = 6 – 12 = **- 6**

10. If α, β, γ are roots of $x^3 - 10x^2 + 6x - 8 = 0$ then find the value of $\alpha^2+\beta^2+\gamma^2$.

Solu : Given α, β, γ are roots of $x^3 - 10x^2 + 6x - 8 = 0$

$S_1 = \alpha + \beta + \gamma = 10$

$S_2 = \alpha\beta + \beta\gamma + \gamma\alpha = 6$

$S_3 = \alpha\beta\gamma = 8.$

$\alpha^2+\beta^2+\gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$

$= (10)^2 - 2(6)$

$= 100 - 12 =$ **88**.

11. Find the transformed equation whose roots are the negatives of the roots of

$x^4 + 5x^3 + 11x + 3 = 0$.

Solu: Let $f(x) = x^4 + 5x^3 + 11x + 3 = 0$.

Required equation is $f(-x) = 0$

$-\alpha_1, -\alpha_2, \ldots\ldots\ldots.. -\alpha_n$ are roots of the equation $f(-x) = 0$

$\Rightarrow (-x)^4 + 5(-x)^3 + 11(-x) + 3 = 0$

$x^4 - 5x^3 - 11x + 3 = 0$.

12. Find the transformed equation whose roots are the reciprocals of the roots of

$x^4 - 3x^3 + 7x^2 + 5x - 2 = 0$.

Solu : Let $f(x) = x^4 - 3x^3 + 7x^2 + 5x - 2 = 0$.

$\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \frac{1}{\alpha_3} \ldots\ldots\ldots.. + \frac{1}{\alpha_n}$ are roots of $f(1/x) = 0$

Required equation is $f(\frac{1}{x}) = 0$

$f(1/x) = (1/x)^4 - 3(1/x)^3 + 7(1/x)^2 + 5(1/x) - 2 = 0$

$\Rightarrow \frac{1 - 3x + 7x^2 + 5x^3 - 2x^4}{x^4} = 0$

$\Rightarrow -2x^4 + 5x^3 + 7x^2 - 3x + 1 = 0$

$\Rightarrow$ **$2x^4 - 5x^3 - 7x^2 + 3x - 1 = 0$.**

13. Find the algebraic equation whose roots are 2 times the roots of

$x^5 - 2x^4 + 3x^3 - 2x^2 + 4x + 3 = 0$.

$2\alpha_1, \ 2\alpha_2, \ \ldots\ldots\ldots .. 2\,\alpha_n$ are roots of the equation $f(x/2) = 0$

Solu : Let $f(x) = x^5 - 2x^4 + 3x^3 - 2x^2 + 4x + 3 = 0$.

Required equation is $f(x/2) = 0$

$\Rightarrow \ (x/2)^5 - 2(x/2)^4 + 3(x/2)^3 - 2(x/2)^2 + 4(x/2) + 3 = 0$

$\Rightarrow \frac{x^5}{32} - 2\left(\frac{x^4}{16}\right) + 3\frac{x^3}{8} - 2\frac{x^2}{4} + 2x + 3 = 0$

$X^5 - 4x^4 + 12x^3 - 16x^2 + 64x + 96 = 0$

14. Find the polynomial equation whose roots are squares of the roots of

$x^3 + 3x^2 - 7x + 6 = 0$

α_1^2, α_2^2________α_n^2 are roots of $f(\sqrt{x}) = 0$

Solu : Let $f(x) = x^3 + 3x^2 - 7x + 6 = 0$

Required equation is $f(\sqrt{x}) = 0$

$\Rightarrow (\sqrt{x})^3 + 3(\sqrt{x})^2 - 7(\sqrt{x}) + 6 = 0$

$\Rightarrow \ x\sqrt{x} + 3x - 7\sqrt{x} + 6 = 0$

$\Rightarrow \sqrt{x}(x - 7) = -(3x+6)$

Squaring both sides

$\Rightarrow \ x(x^2 - 14x + 49) = 9x^2 + 36x + 36$

$\Rightarrow \ X^3 - 14x^2 + 49x = 9x^2 + 36x + 36$

$X^3 - 23x^2 + 13x - 36 = 0$

LONG ANSWER QUESTIONS (7MARKS)

2. Solve the equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

Solu : Given equation is $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0 \ \rightarrow$ **(1)**

which is an even degree reciprocal equation of class -I

Dividing the equation (1) by x^2

$x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0 \implies (x^2 + \frac{1}{x^2}) - 10(x + \frac{1}{x}) + 26 = 0 \rightarrow$ **(2)**

Let $x + \frac{1}{x} = a \implies x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2 \implies x^2 + \frac{1}{x^2} = a^2 - 2$

Substitute in eqn (2)

$\implies a^2 - 2 - 10a + 26 = 0$

$\implies a^2 - 10a + 24 = 0 \implies (a-6)(a-4) = 0 \implies a = 6$ or $a = 4$

Case -I) If a = 6

$x + \frac{1}{x} = 6 \implies x^2 - 6x + 1 = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{6 \pm \sqrt{6^2 - 4.1.1}}{2.1} \implies x = \frac{6 \pm 4\sqrt{2}}{2} = \mathbf{3 \pm 2\sqrt{2}}$

Case - ii) If a= 4

$x + \frac{1}{x} = 4 \implies x^2 - 4x + 1 = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{4 \pm \sqrt{4^2 - 4.1.1}}{2.1} \implies x = \frac{4 \pm \sqrt{12}}{2} = \mathbf{2 \pm \sqrt{3}}$

Hence the roots of the given equation are $3 \pm 2\sqrt{2}$, $2 \pm \sqrt{3}$

3. Solve $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$.

Solu: Given equation is $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$.→ **(1)**

which is an even degree reciprocal equation of class -I

Dividing the equation (1) by x^2

$6x^2 - 35x + 62 - \frac{35}{x} + \frac{6}{x^2} = 0 \implies 6(x^2 + \frac{1}{x^2}) - 35(x + \frac{1}{x}) + 62 = 0 \rightarrow$ **(2)**

Let $x + \frac{1}{x} = a \implies x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2 \implies x^2 + \frac{1}{x^2} = a^2 - 2$

Substitute in eqn (2)

$\implies 6(a^2 - 2) - 35a + 62 = 0$

$\implies 6a^2 - 35a + 50 = 0 \implies 6a^2 - 20a - 15a + 50 = 0$

$\implies (2a-5)(3a-10) = 0 \implies a = 5/2$ or $a = 10/3$

Case -I) If a = 5/2

x + $\frac{1}{x}$ = 5/2 $\Longrightarrow$ $2x^2$ - 5x + 2 = 0 $\Longrightarrow$ $2x^2$ -4x -x + 2 = 0

$\Longrightarrow$ (2x-1) (x-2) = 0 $\Longrightarrow$ **x = 1/2 , x = 2**

Case - ii) If a= 10/3

x + $\frac{1}{x}$ = 10/3 $\Longrightarrow$ $3x^2$ - 10x + 3 = 0 $\Longrightarrow$ $3x^2$ -9x -x + 3 = 0

$\Longrightarrow$ (3x-1) (x-3) = 0 $\Longrightarrow$ **x = 1/3 , x = 3**

Hence the roots of the given equation are 3 , 1/3 , 2 , 1/2 .

4. **Solve the equation x^5 - 5 x^4 + 9 x^3 - 9x^2 + 5x - 1 = 0 .**

Solu: Given equation is x^5 - 5 x^4 + 9 x^3 - 9x^2 + 5x - 1 = 0 . →**(1)**

Which is an odd degree reciprocal equation of class -II

1 is a root of the given equation hence by synthetic division

1	1	-5	9	-9	5	-1
	0	1	-4	5	-4	1
	1	-4	5	-4	1	0

The reduced equation is x^4 - $4x^3$ + $5x^2$ - 4x + 1 = 0 →**(2)**

Clearly eqn (2) is an even degree reciprocal equation of class - I

Dividing the equation (2) by x^2

x^2 - 4x + 5 - $\frac{4}{x}$ + $\frac{1}{x^2}$ = 0 $\Longrightarrow$ (x^2 + $\frac{1}{x^2}$) – 4(x + $\frac{1}{x}$) +5 = 0 →**(3)**

Let x + $\frac{1}{x}$ = a $\Longrightarrow$ x^2 + $\frac{1}{x^2}$ = $(x + \frac{1}{x})^2$ - 2 $\Longrightarrow$ x^2 + $\frac{1}{x^2}$ = a^2 - 2

Substitute in eqn (3)

$\Longrightarrow$ a^2 - 2 - 4a + 5 = 0

$\Longrightarrow$ a^2 - 4 a + 3= 0 $\Longrightarrow$ (a- 3) (a- 1) = 0 $\Longrightarrow$ a = 1 or a= 3

Case -I) If a = 1

x + $\frac{1}{x}$ = 1 $\Longrightarrow$ x^2 - x + 1 = 0 $\Longrightarrow x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x = \frac{1 \pm \sqrt{1^2 - 4.1.1}}{2.1} \Rightarrow \boldsymbol{x = \frac{1 \pm i\sqrt{3}}{2}}$$

Case - ii) If a= 3

$$x + \frac{1}{x} = 3 \Rightarrow x^2 - 3x + 1 = 0 \quad \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{3^2 - 4.1.1}}{2.1} \Rightarrow \boldsymbol{x = \frac{3 \pm \sqrt{5}}{2}}$$

Hence the roots of the given equation are 1 , $\frac{1 \pm i\sqrt{3}}{2}$, $\frac{3 \pm \sqrt{5}}{2}$.

4.Solve $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$

Solu: Given equation is $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$.

Is an even degree reciprocal equation of class -II

Hence 1, -1 are roots of the equation . By synthetic division

6	-25	31	0	-31	25	-6	
-1	0	-6	31	-62	62	-31	6
	6	-31	62	-62	31	-6	0
1	0	-25	37	-25	6		
	6	-25	37	-25	6	0	

The reduced equation is $6x^4 - 25x^3 + 37x^2 - 25x + 6 = 0$ →**(2)**

Clearly eqn (2) is an even degree reciprocal equation of class - I

Dividing the equation (2) by x^2

$6x^2 - 25x + 37 - \frac{25}{x} + \frac{6}{x^2} = 0 \Rightarrow 6\left(x^2 + \frac{1}{x^2}\right) - 25\left(x + \frac{1}{x}\right) + 37 = 0$ →**(3)**

Let $x + \frac{1}{x} = a \Rightarrow x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2 \Rightarrow x^2 + \frac{1}{x^2} = a^2 - 2$

Substitute in eqn (3)

$\Rightarrow 6(a^2 - 2) - 25a + 37 = 0$

$\Rightarrow 6a^2 - 12 \quad - 25a + 37 = 0 \Rightarrow 6a^2 - 25a + 25 = 0$

$\Rightarrow 6a^2 - 15a - 10a + 25 = 0 \Rightarrow 3a(2a-5) - 5(2a-5) = 0$

$\Rightarrow (2a-5)(3a-5) = 0 \Rightarrow a = 5/2$ or $a = 5/3$.

Case -I) If a = 5/2

$x + \frac{1}{x} = 5/2 \Rightarrow 2x^2 - 5x + 2 = 0 \quad \Rightarrow \quad 2x^2 - 4x - x + 2 = 0$

$\Rightarrow \quad (2x-1)(x-2) = 0 \quad \Rightarrow$ **x = 1/2 , x = 2**

Case - ii) If a= 5/3

$x + \frac{1}{x} = 5/3 \Rightarrow 3x^2 - 5x + 3 = 0 \quad \Rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$x = \frac{5 \pm \sqrt{5^2 - 4.3.3}}{2.3} \Rightarrow x = \frac{5 \pm \sqrt{25 - 36}}{6} \Rightarrow \boldsymbol{x = \frac{5 \pm i\sqrt{11}}{6}}$$

Hence the roots of the given equation are 1 , -1, $\frac{5 \pm i\sqrt{11}}{6}$, 2 , 1/2

5. Solve $8x^3 - 36x^2 - 18x + 81 = 0$ given that roots are in A.P.

Solu : Let a-d , a , a+d be the roots of given equation

S_1 = sum of the roots = - P_1 = $-\left(\frac{a_1}{a_0}\right) = \frac{-(-36)}{8}$

$\Rightarrow a - d + a + a + d = \frac{(36)}{8}$

$\Rightarrow 3a = \frac{(36)}{8} \Rightarrow a = \frac{(36)}{24} \Rightarrow$ $\boldsymbol{a = \frac{3}{2}}$

S_3 = product of the roots = - P_3 = $-\left(\frac{a_3}{a_0}\right) = \frac{-(81)}{8}$

$\Rightarrow (a - d) a (a + d) = \frac{-(81)}{8}$

$\Rightarrow a(a^2 - d^2) = \frac{(-81)}{8} \Rightarrow \frac{3}{2}(\frac{9}{4} - d^2) = \frac{(-81)}{8}$

$\Rightarrow (\frac{9}{4} - d^2) = \frac{(-81)}{8} \times \frac{2}{3} \Rightarrow d^2 = \frac{9}{4} + \frac{27}{4} = 9$

$\Rightarrow$ **d = $\pm$ 3**

d = 3 , a= 3/2

Therefore the roots are a-d , a, a+d (i.e) -3/2 , 3/2 , 9/2.

6. Solve $4x^3 - 24x^2 + 23x + 18 = 0$ given that roots are in A.P.

Solu : Let a-d , a , a+d be the roots of given equation

S_1 = sum of the roots = - P_1 = $-\left(\frac{a_1}{a_0}\right) = \frac{-(-24)}{4}$

$\Rightarrow$ a - d + a +a +d = $\frac{(24)}{4}$

$\Rightarrow$ 3a = $\frac{(24)}{4}$ $\Rightarrow$ a = $\frac{(24)}{12}$ $\Rightarrow$ **a = 2**

S_3 = product of the roots = - P_3 = $-\left(\frac{a_3}{a_0}\right) = \frac{-(18)}{4}$

$\Rightarrow$ (a - d) a (a +d) = $\frac{-(18)}{4}$

$\Rightarrow$ a(a^2 - d^2) = $\frac{(-9)}{2}$ $\Rightarrow$ 2(4- d^2) = $\frac{(-9)}{2}$

$\Rightarrow$ (4- d^2) = $\frac{(-9)}{2} \times \frac{1}{2}$ $\Rightarrow$ d^2 = $\frac{9}{4} + 4 = \frac{25}{4}$

$\Rightarrow$ **d** = $\pm\frac{5}{2}$

d = 5/2 , a = 2

Therefore the roots are a-d , a, a+d (i.e) -1/2 , 2, 9/2.

7. Solve $18x^3 + 81x^2 + 121x + 60 = 0$ given that a root is equal to half of the sum of the remaining roots.

Solu : Given equation is $18x^3 + 81x^2 + 121x + 60 = 0$

Given that a root is equal to half of the sum of the remaining roots.

$\Rightarrow$ The roots are in A.P

Let a-d , a , a+d be the roots of given equation

S_1 = sum of the roots = - P_1 = $-\left(\frac{a_1}{a_0}\right) = \frac{-(81)}{18}$

$\Rightarrow$ a - d + a +a +d = $-\frac{(81)}{18}$

$\Rightarrow$ 3a = $\frac{-(81)}{18}$ $\Rightarrow$ a = $\frac{-(81)}{54}$ $\Rightarrow$ **a = $-\frac{3}{2}$**

S_3 = product of the roots = - P_3 = $-\left(\frac{a_3}{a_0}\right) = \frac{-(60)}{18}$

$\Rightarrow$ (a - d) a (a +d) = $\frac{-(10)}{3}$

$\Rightarrow a(a^2 - d^2) = \frac{(-10)}{3} \Rightarrow \frac{3}{2}(\frac{9}{4} - d^2) = \frac{-(10)}{3}$

$\Rightarrow (\frac{9}{4} - d^2) = \frac{(-10)}{3} x - \frac{2}{3} \Rightarrow d^2 = \frac{9}{4} - \frac{20}{9} = 1/36$

$\Rightarrow$ **d = $\pm$ 1/6**

Therefore the roots are a-d , a, a+d

$\Rightarrow$ -3/2 - 1/6, -3/2 , -3/2 + 1/6

$\Rightarrow$ **-5/3 , -3/2 , -4/3**

8. Solve $3x^3 - 26x^2 + 52x - 24 = 0$ given that roots are in G.P.

Solu: Let the roots be a/r , a , ar .

$S_3 = \frac{a}{r} . a . ar = \frac{-(-24)}{3} = 8 \Rightarrow a^3 = 8 \Rightarrow$ **a = 2**

$S_1 = \frac{a}{r} + a + ar = \frac{-(-26)}{3} \Rightarrow \frac{a + ar + ar^2}{r} = \frac{26}{3}$

$\Rightarrow \frac{2 + 2r + 2r^2}{r} = \frac{26}{3} \Rightarrow 6 + 6r + 6r^2 = 26r \Rightarrow 6 - 20r + 6r^2 = 0$

$\Rightarrow 3r^2 - 10r + 3 = 0 \Rightarrow (r-3)(3r-1) = 0 \Rightarrow$ **r= 3 or r = 1/3**

Taking r = 3 and a = 2

The roots are a/r , a , ar $\Rightarrow$ 2/3 , 2 , 6

9. Solve $x^3 - 7x^2 + 14x - 8 = 0$ given that roots are in G.P.

Solu: Let the roots be a/r , a , ar .

$S_3 = \frac{a}{r} . a . ar = -(-8) \Rightarrow a^3 = 8 \Rightarrow$ **a = 2**

$S_1 = \frac{a}{r} + a + ar = -(-7) \Rightarrow \frac{a + ar + ar^2}{r} = 7 \Rightarrow \frac{2 + 2r + 2r^2}{r} = 7$

$\Rightarrow 2r^2 + 2r - 7r + 2 = 0 \Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow 2r^2 - 4r - r + 2 = 0 \Rightarrow (r-2)(2r-1) = 0 \Rightarrow$ **r= 2 or r = 1/2**

Taking r = 2 and a = 2

The roots are a/r , a , ar $\Rightarrow$ 1 , 2 , 4

10. Solve $x^4 + x^3 - 16x^2 - 4x + 48 = 0$ given that product of two roots is 6

Solu: Let $\alpha, \beta, \gamma, \delta$ be the roots of the given equation.

Given that product of two roots is 6 $\Rightarrow \alpha\beta = 6 \rightarrow (1)$

$S1 = \alpha + \beta + \gamma + \delta = -1$

$S2 = \alpha\beta + \beta\gamma + \gamma\delta + \alpha\delta = -16$

$S3 = \alpha\beta\gamma + \beta\gamma\delta + \gamma\delta\alpha + \alpha\beta\delta = 4$

$S4 = \alpha\beta\gamma\delta = 48$

Substituting (1) in S_4 we get $6\gamma\delta = 48 \Rightarrow \boldsymbol{\gamma\delta = 8} \rightarrow (2)$

From $S_3 = \alpha\beta(\gamma + \delta) + \gamma\delta(\alpha + \beta) = 4$

Now substituting (1) and (2) values and from S_1, $\gamma + \delta = -1-(\alpha + \beta)$

$S_3 = 6(-1-(\alpha + \beta)) + 8(\alpha + \beta) = 4$

$\Rightarrow -6 - 6\alpha - 6\beta + 8\alpha + 8\beta = 4$

$\Rightarrow 2\alpha + 2\beta = 10 \Rightarrow \boldsymbol{\alpha + \beta = 5} \rightarrow$ **(3)**

Substituting (3) **in** $\gamma + \delta = -1-(\alpha + \beta)$ we get $\boldsymbol{\gamma + \delta = -6} \rightarrow$ **(4)**

$\alpha + \beta = 5$, $\alpha\beta = 6$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta \Rightarrow (\alpha - \beta)^2 = 25 - 24 = 1$

$\alpha - \beta = 1$ and $\alpha + \beta = 5$ solving $\boldsymbol{\alpha = 3}$ **and** $\boldsymbol{\beta = 2}$

Similarly solving (2) and (4)

$(\gamma - \delta)^2 = (\gamma + \delta)^2 - 4\gamma\delta \Rightarrow (\gamma - \delta)^2 = 36 - 32 = 4$

$\gamma - \delta = 4$ and $\gamma + \delta = -6$ solving $\boldsymbol{\gamma = -2}$ **and** $\boldsymbol{\delta = -4}$

∴The roots are 3, 2 , - 2 , - 4.

11. Solve $x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$ given that it has two pairs of equal roots .

Solu: Given that the equation has two pairs of equal roots

Let $f(x) = x^4 + 4x^3 - 2x^2 - 12x + 9 = 0$

$\Rightarrow f^1(x) = 4x^3 + 12x^2 - 4x - 12$

$\Longrightarrow f^1(x) = 4x^3 + 12x^2 - 4x - 12$

$f(1) = 1 + 4 - 2 - 12 + 9 = 0$

$f^1(1) = 4 + 12 - 4 - 12 = 0$

$\Longrightarrow$ 1 is a multiple root of $f(x) = 0$

By synthetic division

1	1	4	-2	-12	9	
	0	1	5	3	-9	
	1	5	3	-9	0	
1	0	1	6	9		
	1	6	9	0		

The reduced equation is $x^2 + 6x + 9 = 0$

$\Longrightarrow (x+3)^2 = 0 \Longrightarrow$ **$x = -3$, -3**

The roots are 1, 1, -3 , -3 .

12. **Find the polynomial equation whose roots are translates of those of the equation $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ by -2**

Solu: Let $f(x) = x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$.

Required equation is $f(x+2) = 0$.

By synthetic division.

2	1	-5	7	-17	11	
	0	2	-6	2	-30	
2	1	-3	1	-15	-19	$= a_4$
	0	2	-2	-2		
2	1	-1	-1	-17	$= a_3$	
	0	2	2			
2	1	1	1	$= a_2$		

0 2

2 | 1 | 3 = a_1

0

1 = a_0

The required equation is $a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$

$\Longrightarrow$ $x^4 + 3x^3 + x^2 - 17x - 19 = 0$

13. **Find the polynomial equation whose roots are translates of those of the equation** $x^5 + 4x^3 - x^2 + 11 = 0$ **by -3**

Solu: Let $f(x) = x^5 + 4x^3 - x^2 + 11 = 0$

Required equation is $f(x+3) = 0$. By synthetic division.

3 | 1 0 4 -1 0 11

0 3 9 39 114 342

3 | 1 3 13 38 114 | 353 = a_5

0 3 18 93 393

3 | 1 6 31 131 | 507 = a_4

0 3 27 174

3 | 1 9 58 | 305 = a_3

0 3 36

3 | 1 12 | 94 = a2

0 3

3 | 1 | 15 = a_1

3 | 0

1 = a_0

The required equation is $a_0x^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4 + a_5 = 0$

$\Longrightarrow$ $x^5 + 15x^4 + 94x^3 + 305x^2 + 507x + 353 = 0$

14. Transform $x^4 + 4x^3 + 2x^2 - 4x - 2 = 0$ into one in which the coeffecient of second highest power of x axis is zero and also find its transformed equation.

Solu: Given equation is $f(x) = x^4 + 4x^3 + 2x^2 - 4x - 2 = 0$

To eliminate second highest powers of x term in f(x) = 0 we transform

f(x) = 0 into f(x+h) = 0 such that h = $-\left(\frac{a_1}{na_0}\right) = -\left(\frac{4}{4.1}\right) = $ **-1**

The required transformed equation is

$F(x) = 0 \implies a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$

-1	1	4	2	-4	-2	
	0	-1	-3	1	3	
-1	1	3	- 1	-3	1	**= a_4**
	0	-1	-2	3		
-1	1	2	-3	0		= **a_3**
	0	-1	-1			
-1	1	1	-4			= **a_2**
	0	-1				
-1	1	0				= **a_1**
	0					
	1					= **a_0**

The required equation is $f(x-1) = a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$

$\implies$ **$x^4 - 4x^2 + 1 = 0$**

CHAPTER- 5

PERMUTATION & COMBINATION

Weightage : (2 + 2 + 4 + 4)

Key Concepts:

PERMUTATIONS OF DISTINCT THINGS

Fundamental principle of addition: If an event A can happen in m ways, another even B which is independent of A can happen in n ways, then either A or B [exactly one of A, B or only one of A, B] can happen in (m+n) ways.

Ex: In a class 25 girls and 15 boys, the class teacher wants to elect one student as class monitor. So that total number of ways of selecting "exactly one student" monitor either from 25 girls or from 15 boys = 25 + 15 = 40.

Fundamental principle of multiplication [counting principle]:

If an event A can happen in "m" ways and another event B which is independent of A can be happen in m ways, then both events A and B in succession can happen in m x n ways.

Ex: In a college, from a section 3 girls [g_1, g_2, g_3] and 2 boys (b_1, b_2) are willing to join in the invitation committee. The number of ways that the principal can select two students, so that there must be "one boy and one girl" in the invitation committee = 3 x 2 = 6. They are g_1 b_1 g_2 , b_2, g_3 b_3.

$$\Rightarrow {}^nP_r = \frac{n!}{(n-r)!}$$

$\Rightarrow {}^np_r = n(n-1)$ ...... r terms

$\Rightarrow {}^np_n = n!$

$\Rightarrow {}^np_{n-1} = n!$

$\Rightarrow$ 0! = 1, 1! = 1, 2! = 2, 3! = 6

4! = 24, 5! = 120, 6! = 720 and so on

Very Short Answers (2 M)

Level – I

1. If $^{n}p_4$ = 1680 find n

Sol. L.H.S $^{n}p_4$ = n(n - 1) (n - 2) (n - 3)------(1)

R.H.S = 1680 x 10 = (2x84) x (5 x 2) = (2 x 2 x 42) x (5 x 2)

= (2 x 2 x 6 x 7) x (5 x 2) = 8 x 7 x 6 x 5 ……… (2)

$\therefore$ comparing (1) & (2), we get n = 8

2. Find the number of 5 letters words that can be formed using the letters of the word RHYME if each letter can be used any number of times.

Sol: The given word RHYME has 5 letters. The number of 5 letter words that can be formed using the letters of the word RHYME when repetition is allowed = n^r

= 5^5 = 3125.

3. Find the number of injections of a set A with 5 elements to a set B with 7 elements.

Sol: The number of injections from a set containing 75 elements in to a set B with 7 elements.

$^{n}p_m = {}^{7}p_5$ = 7 x 6 x 5 x 4 x 3 = 2520

4. If $^{n}p_7$ = 42. $^{n}p_5$ then find n.

Sol: Given that $^{n}p_7$ = 42. $^{n}p_5$

$\Rightarrow$(n - 5) (n - 6) = 42 $\Rightarrow$(n - 5) (n - 6) = 7 x 6 $\Rightarrow$ n - 5 = 7 $\Rightarrow$n = 7 + 5 = 12

5. In a class there are 30 students. On the New Year day, everystudent posts a greeting card to all his / her classmates. Find the total number of greeting cards posted by them.

Total number of students = 30

Number of greeting cards posted between any 2 student

[say A to B & B to A] = 2

Total number of greeting cards posted by 30 students

$^{n}p_2 = {}^{30}p_2$ = 30 x 29 = 870

6. **If : $^{56}p_{(r+6)} : {}^{54}p_{(r+3)}$ = 30800:1, find r**.

Sol. $$\frac{^{56}p_{(r+6)}}{^{54}p_{(r+3)}} = \frac{30800}{1} \Rightarrow \frac{(56)!}{(56-(r+6)!}\cdot\frac{(54-(r+3))!}{(54)!} = \frac{30800}{1}$$

$$\Rightarrow \frac{(56)!}{(50-r)!}\cdot\frac{(51-r)!}{(54)!} = \frac{30800}{1} \Rightarrow \frac{(56)(55)(54)!}{(50-r)!}\cdot\frac{(51-r)\,(50-r)!}{54!} = 30800$$

$$\Rightarrow 56 \text{ x } 55 \text{ x } (51-r) = 30800 \Rightarrow 51-r = \frac{30800}{56x55} = \frac{308x10x10}{56x55x11} = \frac{77x4x10x10}{7x8x5x11} = 10$$

$\therefore$ 51 – r = 10 $\Rightarrow$r = 51 – 10 = 41

7. **Find the number of 4 letter words that can be formed using the word PISTON in which at least one letter is repeated.**

Sol. The given word PISTON has 6 letters. The number of 4 letters words that can be formed using these 6 letters.

i) When repetition is allowed is = $n^r = 6^4$

ii) When repetition is not allowed = ${}^np_4 = {}^6p_4$

So, the number of 4 letters words with one letter repeated = $6^4 - {}^6p_4$

= 1296 – 360 = 936

8. **If ${}^{12}c_r$ = 495 find r.**

Sol. Given ${}^{12}c_r$ = 495 = 5 x 99 = 11 x 9 x 5 = $\frac{12 \text{ x } 11 \text{ x } 9 \text{ x } 5 \text{ x } 2}{12 \text{ x } 2}$

= $\frac{12 \text{ x } 11 \text{ x } 10 \text{ x } 9}{1 \text{ x } 2 \text{ x } 3 \text{ x } 4}$ = ${}^{12}c_4 \ (or) \ {}^{12}c_8 \left[\because {}^nc_r = {}^nc_{n-r}\right]$

$\therefore$ r = 4(or) 8

9. Find the number of permutations that can be made by using all the letters of the word INDEPENDENCE.

Sol. The given word INDEPENDENCE contains 12 letters.

4 E's, 3 N's, 2 D's

$\therefore$ the required number of arrangements = $\frac{n!}{p!q!r!} = \frac{12!}{4!\,3!\,2!}$

10. **Find the number of different chains that can be prepared using 7 different coloured beads.**

Sol. The number of chains that can be formed using n beads is $\frac{1}{2}(n-1)!$

Hence; the number of chains with 7 different coloured beads is

= $\frac{1}{2}(7-1)! = \frac{1}{2}6! = \frac{1}{2}(720) = 360$

Very Short Answers (2 Marks)

LEVEL - II

1. **Find the number of 4-digited numbers that can be formed using the digits 1, 2, 3, 4, 5, 6 when repetition is allowed.**

Sol. The number of 4-digited number that can be formed using the 6 digits 1, 2, 3, 4, 5, 6 when repletion is allowed $n^r = 6^4$ = 1296.

2. **If ${}^{(n+1)}p_5 : {}^np_6$ = 2 : 7 then find n.**

Sol. Given ${}^{(n+1)}p_5 : {}^np_6$ = 2 : 7 $\Rightarrow$ $2\,{}^np_6 = {}^{7(n+1)}p_5$

2n(n-1) (n-2) (n-3) (n-4) (n-5) = 7(n+1) n(n-1) (n-2) (n-3)

$\Rightarrow$ 2(n-4) (n-5) = 7(n+1) $\Rightarrow 2(n^2-9n+20)=7n+7 \Rightarrow 2n^2-18n+40=7n+77$

$\Rightarrow$ $2n^2 - 25n + 33 = 0 \Rightarrow 2n^2 - 22n - 3n + 33 = 0 \Rightarrow 2n(n-11) - 3(n-11) = 0$

$\Rightarrow$ $(n-11)(2n-3) = 0 \Rightarrow n=11$ (n cannot be a fraction)

3. Find the number of 4-digited numbers that can be formed using the digits 1, 2, 4, 5, 7, 8 when repetition is allowed.

Sol. The number of 4-digit numbers that can be formed using the 6-digits 1, 2, 4, 5, 7, 8 when repetition is allowed is $n^r = 6^4 = 1296$.

4. There are 4 copies alike each of 3 different books. Find the number of ways of arranging these 12 books in a shelf in a single row.

Sol. The total number of books = 3 x 4 = 12.

Among 12 books: 4 books are alike of one kind, 4 books are alike of second kind and 4 books are alike of third kind.

$\therefore$ the required number of ways = $\frac{n!}{p!\,q!\,r!} = \frac{12!}{4!\,4!\,4!}$

5. Find the number of ways of arranging 4 boys and 3 girls around a circle so that all the girls sit together.

Sol. Treat all the 3 girls as one unit. Then we have 4 boys and 1 unit of girls.

These 5 can be arranged around a circle in (5-1)! = 4! Ways.

Now, the 3 girls can be arranged among themselves in 3! Ways

$\therefore$ the required number of arrangements = 4! X 3! = 24 x 6 = 144.

6. If ${}^{15}c_{2r-1}$ = ${}^{15}c_{2r+4}$ then find r.

Sol. ${}^{15}c_{2r-1} = {}^{15}c_{2r+4}$ then find r. $\left[\because {}^{n}c_r = {}^{n}c_s \Rightarrow r+s = n\,(or) r = s\right]$

$\Rightarrow 4r + 3 = 15 \Rightarrow 4r = 12 \Rightarrow r = 3$

7. If ${}^{n}p_r$ = 5040, ${}^{n}c_r$ = 210 then find n and r

Sol. We know that $\frac{{}^{n}p_r}{{}^{n}c_r} = r!$

$\Rightarrow \frac{{}^{n}p_r}{{}^{n}c_r} = \frac{5040}{210}$ = 24 = 4 x 3 x 2 x 4 x 1 = 4! = r!

$\therefore$ r = 4

Now ${}^{n}p_4$ = 5040 = x 10 x 504 = 10 x 9 x 56

= 10 x 9 x 8 x 7 = ${}^{10}p_4$ [$\therefore$n=0]

$\therefore$ n = 10, r = 4

8. Find the number of ways of selecting 7 numbers from a contingent of 10 soldiers.

Sol. The number of ways of selecting 7 numbers out 10 soldiers

${}^{10}c_7 = \frac{10!}{3!\,7!}$ = 120

9. Find the number of 5 letter words that can be formed using the letters of word MIXTURE which begin with a vowel when repetitions are allowed.

Sol: We have to fill up 5 blanks using the letter of word MIXTURE having 7 letters among them 3 are vowels. Fill the first place with one of the vowels [I (or) U(or) E] in 3 ways.

x				

Each of the remaining 4 places can be filled in 7 way. Thus the number of 5-letter words is 3 x 7 x 7 x 7 x7 = 3×7^4

10.Find the number of ways of arranging the letters of the word

I) MATHEMATICS ii) INTERMEDIATE

Solu : I) The number of linear permutations of n things in which there are p things of one kind , q things alike of one kind and r things alike of another kind is $\frac{n!}{p!q!r!}$

The word MATHEMATICS has 11 letters .

It has 2 M's , 2 A's ,2 T's, and 1 H, 1 E , 1 I , 1 C, 1 S

Hence number of ways = $\frac{11!}{2!2!2!}$

Ii) The word INTERMEDIATE has 12 letters .

It has 2 I's , 3 E's ,2 T's, and 1 R, 1 N , 1 M , 1 D, 1 A

Hence number of ways = $\frac{12!}{2!3!2!}$

1. **Find the number of positive divisors of 1080.**

Solu: $1080 = 2^3 \times 3^3 \times 5^1$

Number of positive divisors of $n = a^p\ b^q\ c^r$ is $(p+1)(q+1)(r+1)$

The number of positive divisors of 1080 = (3+1)(3+1)(1+1) = 32

2. Find the number of diagonals of a polygon with 12 sides.

Solu : Number of diagonals of a polygon with n sides is $^nC_2 - n$

Hence number of diagonals of a polygon with 12 sides

Is $^{12}C_2 - 12 = 66 - 12 = 44$

Short Answer Questions:

Level : 1

1. Simplify $^{34}C_5 + \sum_{r=0}^{4} {}^{38-r}C_4$

Solu : $^{34}C_5 + \sum_{r=0}^{4} {}^{38-r}C_4$

$= {}^{34}C_5 + {}^{38}C_4 + {}^{37}C_4 + {}^{36}C_4 + {}^{35}C_4 + {}^{34}C_4$ (re grouping terms)

$= ({}^{34}C_5 + {}^{34}C_4) + {}^{35}C_4 + {}^{36}C_4 + {}^{37}C_4 + {}^{38}C_4$

$= ({}^{35}C_5 + {}^{35}C_4) + {}^{36}C_4 + {}^{37}C_4 + {}^{38}C_4$

$= ({}^{36}C_5 + {}^{36}C_4) + {}^{37}C_4 + {}^{38}C_4$

$= ({}^{37}C_5 + {}^{37}C_4) + {}^{38}C_4$

$= ({}^{38}C_5 + {}^{38}C_4) = {}^{39}C_5$

$^nC_{r+1} + {}^nC_r = {}^{n+1}C_{r+1}$

2. Prove that $\frac{^{4n}C_{2n}}{^{2n}C_n} = \frac{1.3.5.......(4n-1)}{\{1.3.5 .....(2n-1)\}^2}$

Solu: $\frac{^{4n}C_{2n}}{^{2n}C_n} = \frac{\frac{4n!}{(4n-2n)!\,2n!}}{\frac{2n!}{(2n-n)!\,n!}} = \frac{\frac{4n!}{(2n)!\,2n!}}{\frac{2n!}{(n)!\,n!}} = \frac{4n!}{(2n!)^2} \; x \; \frac{(n!)^2}{2n!}$

$= \frac{(4n(4n-1)(4n-2)... ....5.4.3.2.1.}{\{(2n(2n-1)(2n-2)........5.4.3.2.1.\}^2} \text{ x } \frac{(n!)^2}{2n!}$

$$= \frac{((4n-1)(4n-3)\dots\dots\dots 5.3.1.)\ (4n(4n-2)\dots 4.2))}{\{((2n-1)(2n-3)\dots\dots 5.3.1)((2n\ (2n-2)\ \dots\dots 4.2.\}^2} \times \frac{(n!)^2}{2n!}$$

$$= \frac{(4n-1)(4n-3)\dots\dots\dots 5.3.1.)\ .(2.2n).[2(2n-1)]\dots(2.2).(2.1])}{\{(2n-1)(2n-3)\dots\dots 5.3.1)\}^2\{(2n\ 2(n-1)\ \dots\dots(2.2).(2.1).\}^2} \times \frac{(n!)^2}{2n!}$$

$$= \frac{(4n-1)(4n-3)\dots\dots\dots 5.3.1.)\,2^{2n}(2n!)}{\{(2n-1)(2n-3)\dots\dots 5.3.1)\}^2\,2^{2n}(n!)^2} \times \frac{(n!)^2}{2n!}$$

$$= \frac{(4n-1)(4n-3)\dots\dots\dots 5.3.1.)}{\{(2n-1)(2n-3)\dots\dots 5.3.1)\}^2}$$

$$= \frac{1.3.5\dots\dots(4n-3)(4n-1)}{\{1.3.5\ \dots(2n-1)\}^2} = \text{R.H.S}$$

3. Find the number of ways of forming a committee of 5 persons from a group of 5 Indians and 4 Russians such that there are a6least 3 Indians in the committee.

Solu : The committee can have 3 Indians, 2Russians or 4 Indians,1 Russian of all 5 Indians.

The number of ways of forming a committee with

R. No.	5 Indians	4 Russians	No.of selections
1.	3	2	$^4C_3 \times {}^4C_2 = 10 \times 6 = 60$
2.	4	1	$^5C_4 \times {}^4C_1 = 5 \times 4 = 20$
3.	5	0	$^5C_5 \times {}^4C_0 = 1 \times 1 = 1$

Number of ways forming a committee of 5 members with atleast 3 Indians is 60 + 20 + 1 = 81.

4. Find the number of numbers that are greater than 4000 which can be formed using the digits 0 , 2, 4, 6, 8 without repetition .

Solu : Given digits { 0 , 2, 4, 6, 8 }

The number of numbers greaterthan 4000 which can be formed using the digits

0 , 2, 4, 6, 8 is

Case : 1. 4- digit number _ _ _ _

4, 6, 8

The first place cannot be filled with 0 , 2. It must be filled with 4, 6, 8

In 3 ways.

Remaining 3 places can be filled with remaining 4 digits in $^4P_3$ ways.

Number of arrangements = 3 x $^4P_3$ = 3 x 4 x 3 x 2 = 72

Case : 2. 5- digit number _ _ _ _ _

2, 4, 6, 8

The first place cannot be filled with 0 . It must be filled with 2, 4, 6, 8

In 4 ways.

Remaining 4 places can be filled with remaining 4 digits in $^4P_4$ ways.

Number of arrangements = 4 x $^4P_4$ = 4 x 24 = 96

Total number of arrangements greater than 4000 is = 72 + 96 = 168.

5. Find the sum of all 4 digit numbers that can be formed using the digits 1, 3, 5, 7, 9.

(without repetition)

Solu : Given digits { 1, 3, 5, 7, 9} , r = 4 , n = 5.

Sum of all r digited numbers = $^{n-1}P_{r-1}$ (sum of n digits) (111...r times)

Sum of all 4 digit numbers = $^{n-1}P_{r-1}$ (sum of n digits) (111...r times)

= $^{5-1}P_{4-1}$ (1 + 3 + 5 + 7 + 9) (1111)

= $^4P_3$ (25) (1111) = 24 x 25 x 1111

= 6,66,600

6. **Out of 7 gents and 5 ladies how many 6 member committees can be formed , such that there will be atleast 3 ladies in the committee.**

Solu : Given there are 7 gents and 5 ladies.

A committee is formed with6 members with atleast 3 ladies in a committee

The number of ways of forming a committee with

S.No.	7 Gents	5 Ladies	No.of selections
1.	3	3	${}^7C_3 \times {}^5C_3 = 35 \times 10 = 350$
2	2	4	${}^7C_2 \times {}^5C_4 = 21 \times 5 = 105$
3	1	5	${}^7C_1 \times {}^5C_5 = 7 \times 1 = 7$

Total number of ways forming a committee is 350 + 105 + 7 = 462

7. If the letters of the word EAMCET are permuted in all possible ways and if the words thus formed are arranged in the dictionary order, find the rank of the word EAMCET.

Sol. The dictionary order of the letters of the word EAMCET is as follows: A, C, E, E, M, T

The number of words that begins with A __ __ __ __ __ $\frac{5!}{2!} = 60$

The number of words that begins with C __ __ __ __ __ $\frac{5!}{2!} = 60$

The number of words that begins with EAC __ __ __ 3! = 6

The number of words that begins with EAE __ __ __ 3! = 6

The number of words that begins with EAMCET = 0!= 1

Hence the rank of the word EAMCET = 60 + 60 + 6 + 6 + = 133

8. Prove that for $3 \leq r \leq n$; ${}^{(n-3)}c_r + 3\,{}^{(n-3)}c_{r-1} + 3\,{}^{(n-3)}c_{r-2} + {}^{(n-3)}c_{r-3} = {}^nc_r$

Sol. We know that ${}^nc_r + {}^nc_{r-1} = {}^{(n+1)}c_r$

LHS = ${}^{(n-3)}c_r + 3.{}^{(n-3)}c_{r-1} + 3\,{}^{(n-3)}c_{r-2} + {}^{(n-3)}c_{r-3}$ [on rewriting terms]

$= \left[{}^{(n-3)}c_r + {}^{(n-3)}c_{r-1}\right] + 2\left[{}^{(n-3)}c_{r-1} + {}^{(n-3)}c_{r-2}\right] + \left[{}^{(n-3)}c_{r-2} + {}^{(n-3)}c_{r-3}\right]$

$= {}^{(n-3+1)}c_r + 2.\,{}^{(n-3+1)}c_{r-1} + {}^{(n-3+1)}c_{r-2}$

$= {}^{(n-2)}C_r + 2.{}^{(n-2)}C_{r-1} + {}^{(n-2)}C_{r-2}$

$= \left[{}^{(n-2)}C_r + {}^{(n-2)}C_{r-1}\right] + \left[{}^{(n-2)}C_{r-1} + {}^{(n-2)}C_{r-2}\right]$

$= {}^{(n-2+1)}C_r + {}^{(n-2+1)}C_{r-1} = {}^{(n-1)}C_r + {}^{(n-1)}C_{r-1}$

$= {}^{(n-1+1)}C_r = {}^{n}C_r$ = R.H.S

9. If the letters of the word "MASTER" are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word "MASTER"

Sol. The dictionary order of the letters of the word "MASTER" are

A, E, M, R, S, T

The number of words that begins with A __ __ __ __ __ 5! = 120

The number of words that begins with E __ __ __ __ __ 5! = 120

The number of words that begin with MAE _ _ _ = 3! = 6

The number of words that begin with MAR _ _ _ = 3! = 6

The number of words that begin with MASE _ _ = 2! = 4

The number of words that begin with MASR _ _ = 2! = 4

The number of words that begin with MASTER = 1

Rank of the word MASTER is 2(5!) + 2(3!) + 2(2!) + 1

= 2(120) + 2 (6) +2(2) +1 = 257

10. If the letters of the word "PRISON" are permuted in all possible ways and the words thus formed are arranged is the dictionary order, then find the rank word "PRISON"

Sol. The dictionary order of the letters of the word PRISON

I, N, O, P, R, S

The number of words that begin with I __ __ __ __ __ 5! = 120

The number of words that begin with N __ __ __ __ __ 5! = 120

The number of words that begin with O __ __ __ __ __ 5! = 120

The number of words that begin with P I __ __ __ __ 4! = 24

The number of words that begin with P N __ __ __ __ 4! = 24

The number of words that begin with P O __ __ __ __ 4! = 24

The number of words that begin with PRIN __ __ 2! = 2

The number of words that begin with PRIO __ __ 2! = 2

The number of words that begin with PRISN __ __ 1! = 2

The number of words that begin with PRISON __ __ 0! = 1

This is required word

$\therefore$ Rank of the word PRISON=3(120) + 3(24) + 2(2) + 1 + 1

= 360 + 72 + 4 + 1 + 1 = 438

11. **Find the number of 4 letter words that can be formed using the letters of the word MIRACLE. How many of them (i) begin with a vowel (ii) begins and end with vowels (iii) end with a constant.**

Sol. The total number of letters in the word MIRACLE is "7" hence the number of 4 letters words ${}^{7}p_{4}$ = 7 x 6 x 5 x 4 = 840. In the word MIRACLE the no. of vowels is 3[I, A, E] and the no. of constants is 4(M, R, C, L)

I) 4 letter words beginning with a vowel:

Number of ways of filling first place with a vowel.

= ${}^{3}p_{1}$ = 3 ☐ ☐ ☐

Number of ways filling the remaining 3 places with the remaining 6 letters [4 consonants + 2 vowels] is ${}^{6}p_{3}$ = 120 from the counting principle, the number of 4 letter words that begin with a vowel is 3 x 120 = 360.

i) **Words beginning and ending with a vowel**:

The number of ways of filling first and last place with 3 vowels is ${}^{3}p_{2}$ =6, number of ways filling the remaining 2 places with remaining 5 letters is ${}^{5}p_{2}$ = 20.

ii) **Words ending with a consonant:**

The number of ways of filling last place with one of the 4 consonants is ${}^{4}p_{1}$ = 4

Number of ways of filling remaining 3 places the remaining 6 letters

${}^{6}p_{3}$ = 6 x 5 x 4 = 120. Thus, the number of 4 letter words that end with a consonant is 4 x 120 = 480.

SHORT ANSWERS

Level-II

1) Find the number of ways arranging the 8 men and 4 women around a circular table. In how many of them (i) all the women come together (ii) no two women come together.

Sol. Total number of persons = 12, (8 men + 4 women)

$\therefore$ The number of circular permutations = (n-1)! = (12-1)! = (11)!

I) treat the 4 women as single unit, then we have 1 unit of women and 8 men = 9 units.These 9 units can be arranged around a circle table in (9-1)!=8! the women among themselves can be arranged in 4! Ways. Hence the required number of arrangements is 8! x 4!.

Ii)First we fix the positions of 8 men

They can be arranged around circular table in (8-1)!=7!

Now the 4 women can be arranged in the remaining 8 gaps in $^{8}p_{4}$ ways

Hence the total number of circular arrangements = 7! x $^{8}p_{4}$

2.Find the number of ways in which 4 letters can be put in 4 addressed envelopes so that no letter goes into the envelope meant for it.

Sol. Formula: number of derangements of n distinct things

$$= n!\left[\frac{1}{2i}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}........+(-1)^{n}\frac{1}{n!}\right]$$

Required number of derangement

$$= 4!\left[\frac{1}{2i}-\frac{1}{3!}+\frac{1}{4!}\right] = 24\left[\frac{1}{2}-\frac{1}{6}+\frac{1}{24}\right] = 24\left[\frac{12-4+1}{24}\right] = 9$$

3. Find the number of 5 letter words that can be formed using the letters of the word EXPLAIN, that begin and end with a vowel when repetitions are allowed.

Sol. The word EXPLAIN has 7 letters, and these are 5 vowels [A, I, E]. since repetition is allowed, he first and last place can be filled by 3 vowels in 3^2 ways.

Since, repetition is allowed, each of the remaining 3 places can be filled in 7^3 ways. Thus the required number of words = $3^2 \times 7^3$=9x343 = 3087.

4 . **Find the number of 5-digit numbers can be formed using the digits 0, 1, 1, 2, 3.**

Sol. The number of all possible 5-digited numbers taken from 0, 1, 1, 2, 3 = $\frac{5!}{2!}$ = 60

But these 60 numbers include the numbers which begin with 0, which are not actually 5 digit numbers. The number of numbers begin with zero taken from 1, 2, 2, 3 = $\frac{4!}{2!}$ = 12.

∴ The required number of numbers = 60 – 12 = 48.

5. Find the number of ways of selecting 11 number cricket team from 7 batsmen, 6 bowlers and 2 wicket keepers so that the contains 2 wicket keepers and at least 4 bowlers.

Solu : Can be selected in the following compositions.

Keepers (2)	Bowlers (6)	Batsmen (7)	No. of selections
2	4	5	$^2c_2 \times {}^6c_4 \times {}^7c_5$ = 1 x 15 x 21 = 315
2	5	4	$^2c_2 \times {}^6c_5 \times {}^7c_4$ = 1 x 6 x 35 = 210
2	6	3	$^2c_2 \times {}^6c_6 \times {}^7c_3$ = 1 x 1 x 35 = 35

∴ The total number of selections= 315 + 210 + 35 = 560.

6 . **9 different letters of an alphabet are given. Find the number of 4 letter words that can be formed using these 9 letters which have (i) no letter is repeated (ii) at least one letter is repeated.**

Sol. (i) The number of 4 letters words that can be formed using the 9 different letters in which no letter is repeated $^np_r = {}^9p_4$ = 9 x 8 x 7 x 6 = 3024

(ii) The number of 4 letters words that can be formed using the 9 different letters in which at least one letter is repeated

$= n^r - {}^np_r = 9^4 - {}^9p_4 = 6561 – 3024 = 3537.$

7 . **Find the number of all 4 letter words that can be formed using the letters of the word EQUATION. How many of these words begin with E? How many end with N? How many begin with E and end with A?**

Sol. (i) the given word EQUATION contains 8 letters. So, that number of 4 letter word formed from it = ${}^8p_4$= 8 x 7 x 6 x 5 = 1680

(ii) **4 letter words beginning with E :**

Fill the first place with E.

Then the remaining 3 places can be filled with the remaining 7 letters in

${}^7p_3$ = 7 x 6 x 5 = 210 ways

(iii) **4 letter words ending with N:**

Fill the last place with N. Then the remaining 3 places can be filled with the remaining 7 letters in ${}^7p_3$ = 7 x 6 x 5 = 210 ways

iv)**4 Letter words beginning with E and ending with A.**

Fill the first place with E and Last place with A.

E			A

Then the remaining 2 places can be filled with remaining 6 letters in

= 6 x 5 = 30 ways.

8. A candidate is required to answer 6 out 10 questions, which are divided into two groups A and B each containing 5 questions. He is not permitted to attempt more than 4 questions from either group. Find the number of different ways in which the candidate can choose six questions.

Sol. The number of ways of answering the questions is possible in the following composition.

Group A(5)	*Group B(5)*	*No. of selections*
4	2	${}^5C_4$ x ${}^5C_2$ = 5 x 10 = 50
3	3	${}^5C_3$ x ${}^5C_3$ = 10 x 10 = 100

2	4	${}^5c_2$ x ${}^5c_4$ = 10 x 5 = 50

$\therefore$ The required number of ways= 50 + 100 + 50= 200

9.**Find the numbers can be formed using all 4-letter words that can be formed using the letters of the word RAMANA.**

Sol. The given word RAMANA has 6 letters with 3A's are alike and rest are different. Now we have to form 4 letter words using these 6 letters.

Here 3 cases are:

Case(i) : All different letters R, A, M, N

The number of 4 letter words formed from R, A, M, N = 4! = 24

Case (ii) : Two like letter A, A and two different from R, M, N.

The two different letters can be choose from R, M, N in ${}^3c_2$ = 3 ways.

$\therefore$ Number of 4 letter words like AARM= 3 x $\frac{4!}{2!}$ = 3 x 12 = 36

Case (iii): Three like letters A, A, A and one from R, M, N

One letter can be chooses from 3 different letters in 3 c_1 = 3 ways

$\therefore$ Number of 4 letter like A, A, A, R= 3 x $\frac{4!}{2!}$ = 3 x 4 = 12

$\therefore$ The required number of 4 letter words = 24 + 36 + 12 = 72

Note: The required number of 6 letter words from RAMANA = $\frac{6!}{3!}$

10. **Find the number of zeros in 100!**

Sol. 100! = $2^{\alpha} 3^{\beta} 5^{\gamma} 7^{\delta}$...... here

$$\alpha = \left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] + \left[\frac{100}{2^3}\right] + \left[\frac{100}{24}\right] + ...$$

= 50 + 25 + 12 + 6 + 3 + 1

= 97

$$\gamma = \left[\frac{100}{2}\right] + \left[\frac{100}{2^2}\right] = 20 + 4 + 24$$

Thus 2 occurs 97 times and 5 occurs 24 times in 100!

To get a 10 we require a 2 and a 5.

Out of ninety seven 2’s we take twenty four 2’s to join with twenty four 5’s

Hence, the number of zeros in 100!= number of 10’s in 100!= 24

Now, the number of zeros in 100! is 24 [since 10 = 2 x 5]

CHAPTER-6
BINOMIAL THEOREM
Weightage: (2 + 7 + 7)

Key Concepts:

Binomial theorem for integral index:

$$(x+y)^n = {}^nC_0x^n + {}^nC_1x^{n-1}y + {}^nC_2x^{n-2}y^2 + ...+ {}^nC_rx^{n-2}y^2 + ...+ {}^nC_ny^n, n \in N$$

The general form of the binomial expansion is

$$T_{r+1} = {}^nC_rx^{n-r}y^r$$

Standard Binomial expansion

$$(1+x)^n = 1+nx+{}^nC_2x^2+.....+{}^nC_rx^r+....+x^n =$$

$$C_0+C_1x+C_2x^2+....+C_rx^r+.....+C_nx^n$$

In the expansion of $(1+x)^n$

i) the coefficient of x^r is ${}^nC_r$ = the coefficient of $(r+1)^{th}$ term

ii) the coefficient of r^{th} term is ${}^nC_{r-1}$

Middle term(s):

i) If n is even then the middle term in the expansion of $(1+x)^n$ is $T_{\frac{n}{2}}+1$

ii) If n is odd then the two middle term are $T_{\frac{n+1}{2}}, T_{\frac{n+3}{2}}$

In $(1+x)^n$ the coefficient of the middle term is the largest Binomial coefficient

Number of terms in the Trinomial expansion of $(x+y+z)^n = \frac{(n+1)(n+2)}{2}$

Very Short Answer Questions **(2Marks)**

1) Find the 3rd term from the end in the expansion of $\left[x^{\frac{-2}{3}} - \frac{3}{x^2}\right]^8$

Sol: 3rd term from the end in $\left[x^{\frac{-2}{3}} - \frac{3}{x^2}\right]^8$ is equal to the 3rd term in $\left[-\frac{3}{x^2} + x^{\frac{-2}{3}}\right]^8$

$\therefore T_3=T_{2+1}={}^8C_2\left[-\frac{3}{x^2}\right]^{8-2}\left[x^{\frac{-2}{3}}\right)^2={}^8C_2\left[\frac{3}{x^2}\right]^6\left[\frac{1}{x^{\frac{2}{3}}}\right]^2$

$T_{r+1} = {}^nc_r \cdot x^{n-r} \cdot y^r$

$=\frac{8\times7}{2}\times3^6\times\frac{1}{x^{12}}\times\frac{1}{x^{\frac{4}{3}}}$

$= 28\times3^6\times\frac{1}{x^{\frac{40}{3}}}$

2) Find the 5th term in the expansion of $(3x-4y)^7$.

Sol: General term of $(x+y)^n$ is $T_{r+1}={}^nC_r x^{n-r}y^r$

$\therefore T_5=T_{4+1}={}^7C_4(3x)^{7-4}(-4y)^4=(35)(27)x^3(256)y^4$

$=241920x^3y^4$

3) Find the middle term (S) in the expansion of $\left[\frac{3x}{7}-2y\right]^{10}$

Sol: Given binomial exponent n=10 is even

$\therefore$ Middle term is $\frac{T_{10}}{2}+1=T_{5+1}=T_6$

$\therefore T_6=T_{5+1}={}^{10}C_5\left[\frac{3x}{7}\right]^{10-5}(-2y)^5$

$=-{}^{10}C_5\left[\frac{3}{7}\right]^5.x^5.2^5.y^5=-{}^{10}C_5\left[\frac{6}{7}\right]^5.x^5.y^5$

4) Find the number of terms in expansion of $(2x+3y+z)^7$

Sol: Number of terms in the trinomial expansion of $[x+y+z]^n=\frac{(n+1)(n+2)}{2}$

$\therefore$ Number of terms in the expansion of $(2x+3y+z)^7=\frac{(7+1)(7+2)}{2}=\frac{8\times9}{2}=36$

5) If A and B are the coefficients of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n+1}$ respectively, then find the value of $\frac{A}{B}$.

Sol: Coefficient of x^n in the expansion of $(1+x)^{2n}$ is $A={}^{2n}C_n$

Coefficient of x^n in the expansion of $(1+x)^{2n-1}$ is $B={}^{2n-1}C_n$

$$\therefore \frac{A}{B}=\frac{^{2n}C_n}{^{(2n-1)}C_n}=\frac{(2n)!}{(n!)(n!)}\times\frac{n!(n-1)!}{(2n-1)!}=\frac{(2n)(2n-1)!}{(n!)n(n-1)!}\times\frac{n!(n-1)!}{(2n-1)!}=\frac{2n}{n}=2$$

6) If $^{22}C_r$ is the largest binomial coefficient in the expansion of $(1+x)^{22}$, Find the value of $^{13}C_r$

Sol: Given binomial exponent n=22 is even

$\therefore$ Largest binomial coefficient is $^nC_{\frac{n}{2}}=\frac{^{22}C_{22}}{2}={}^{22}C_{11}$

Now, $^{22}C_r={}^{22}C_{11}\Rightarrow r=11\ \therefore{}^{13}C_r={}^{13}C_{11}\ C_2=\frac{13\times12}{2\times1}=78$

7) Find the term independent of x in the expansion of $\left[\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right]^{10}$

Sol: General term of $\left[\sqrt{\frac{x}{3}}+\frac{2}{2x^2}\right]^{10}$ is $T_{r+1}={}^{10}C_r\left[\sqrt{\frac{x}{3}}\right]^{10-r}\left[\frac{3}{2x^2}\right]^r$

$$^{10}C_r\left[\frac{1}{3}\right]^{\frac{10-r}{2}}\left[\frac{3}{2}\right]^r\left[x^{\frac{10-r}{2}}\right]\left[\frac{1}{x^2}\right]^r={}^{10}c_r\left[\frac{1}{3}\right]^{\frac{10-r}{2}}\left[\frac{3}{2}\right]^r.x^{\frac{10-r}{2}}.x^{-2r}$$

$$^{10}C_r\left[\frac{1}{3}\right]^{\frac{10-r}{2}}\left[\frac{3}{2}\right]^r.x^{\frac{10-r}{2}-2r}\ ....(1)$$

To get the term independent of x, we put $\frac{10-r}{2}-2r=0\Rightarrow\frac{10-r}{2}=2r\Rightarrow 10-r=4r\Rightarrow 5r=10\Rightarrow r=2$

From (1) the term independent of x is $^{10}C_2\left[\frac{1}{3}\right]^{\frac{10-r}{2}}\left[\frac{3}{2}\right]^2=\frac{10\times9}{2\times1}\left[\frac{1}{3}\right]^1\left[\frac{3^2}{2^2}\right]=\frac{5}{4}$

8) Find the set E of x for which the binomial expansion $[3-4x]^{\frac{3}{4}}$ is valid

Sol: G.E. = $[3-4x]^{\frac{3}{4}}=3^{\frac{3}{4}}\left[1-\frac{4x}{3}\right]^{\frac{3}{4}}$

This is valid when $\left|\frac{4x}{3}\right|<1$

$$\Rightarrow|x|<\frac{3}{4}\Rightarrow x\in\left[\frac{-3}{4},\frac{3}{4}\right]$$

$\therefore E=\left[\frac{-3}{4},\frac{3}{4}\right]$

Long Answer Questions **(7 Marks)**

1) If the coefficient of 4 consecutive terms in the expansion of $(1+x)^n$ are a_1,a_2,a_3,a_4 respectively, then show that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$

Sol: Let the coefficients of 4 consecutive terms of $(1+x)^n$ be $a_1={}^nC_r,\ a_2={}^nC_{r+1}$, $a_3={}^nC_{r+2},\ a_4={}^nC_{r+3}$

L.H.S. = $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{{}^nC_r}{{}^nC_r+{}^nC_{r+1}}+\frac{{}^nC_{r+2}}{{}^nC_{r+2}+{}^nC_{r+3}}$

$=\frac{{}^nC_r}{{}^{(n+1)}C_{r+1}}+\frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}}\left[\because {}^nC_r+{}^nC_{r+1}={}^{(n+1)}C_{r+1}\right]$

$=\frac{{}^nC_r}{\left(\frac{n+1}{r+1}\right){}^nC_r}+\frac{{}^nC_{r+2}}{\left(\frac{n+1}{r+3}\right){}^nC_{r+2}}\left[\because {}^nC_r=\left(\frac{n}{r}\right){}^{n-1}C_{r-1}\right]$

$=\frac{r+1}{n+1}+\frac{r+3}{n+1}=\frac{r+1+r+3}{n+1}=\frac{2r+4}{n+1}=\frac{2(r+2)}{n+1}$.......(1)

R.H.S. = $\frac{2a_2}{a_2+a_3}=\frac{2\left({}^nC_{r+1}\right)}{{}^nC_{r+1}+{}^nC_{r+2}}=\frac{2\left({}^nC_{r+1}\right)}{{}^{(n+1)}C_{r+2}}=\frac{2\left({}^nC_{r+1}\right)}{\left(\frac{n+1}{r+2}\right)\left({}^nC_{r+1}\right)}$

$=\frac{2}{\frac{n+1}{r+2}}=\frac{2(r+2)}{n+1}$...............(2)

From (1) and (2) L.H.S. = R.H.S.

2) If C_r denotes ${}^nC_r$ then prove that $C_0+C_1.\frac{x}{2}+C_2.\frac{x^3}{3}+......+C_n.\frac{x^n}{n+1}=\frac{(1+x)^{n+1}}{(n+1)x}$ **. Also deduce that** $C_0+\frac{C_1}{2}+\frac{C_2}{3}+......+\frac{C_n}{n+1}=\frac{2^{n+1}-1}{n+1}$

Sol: **Method-I** :

Let S = $C_0+C_1.\frac{x}{2}+C_2.\frac{x^2}{3}+.......+C_n.\frac{x^n}{n+1}={}^nC_0+{}^nC_1\frac{x}{2}+{}^nC_2\frac{x^2}{3}+.....+{}^nC_n.\frac{x^n}{n+1}$

$\Rightarrow x.s={}^nC_0.x+{}^nC_1\frac{x^2}{2}+{}^nC_2.\frac{x^3}{3}+......+{}^nC_n.\frac{x^{n+1}}{n+1}$

$$\Rightarrow (n+1)xS=\frac{n+1}{1}.{}^{n}C_0x+\frac{n+1}{2}.{}^{n}C_1.x^2+\frac{n+1}{3}.{}^{n}C_2.x^3+.....+\frac{n+1}{n+1}\ {}^{n}C_n.x^{n+1}$$

$$= {}^{n+1}C_1.x+{}^{n+1}C_2x^2+{}^{n+1}C_3x^3+......+{}^{n+1}C_{n+1}.x^{n+1}\left[\because \left[\frac{n+1}{r+1}\right]\ {}^{n}C_r={}^{(n+1)}C_{r+1}\right]$$

$$\Rightarrow (n+1)xS=(1+x)^{n+1}-1\left[\because\ {}^{n}C_1x+\ {}^{n}C_2x^2+.....+\ {}^{n}C_nx^n=(1+x)^n-1\right]$$

$$\Rightarrow S=\frac{(1+x)^{n+1}-1}{(n+1).x}$$

Method-II :

LHS = $C_0+\frac{C_1}{2}x+\frac{C_2}{3}x^2+.......+\frac{C_n}{n+1}=\ {}^{n}C_0+\frac{{}^{n}C_1}{2}x+\frac{{}^{n}C_2}{3}x^2+.....+\frac{{}^{n}C_n}{n+1}x^n$

$$=1+\frac{n}{(1)^2}x+\frac{n(n-1)}{(1.2)3}x^2+.....+\frac{1}{n+1}x^n$$

$=\frac{1}{(n+1)^n}\left[(n+1)x+\frac{(n+1)nx^2}{1.2}+\frac{(n+1)n(n-1)}{1.2.3}x^3+......+x^{n+1}\right]$ (Multiplying and Dividing by (n+1)x)

$$=\frac{1}{(n+1)x}\left[{}^{n+1}C_1x+\ {}^{n+1}C_2x^2+\ {}^{n+1}C_3x^3+......+\ {}^{n+1}C_{n+1}x^{n+1}\right]$$

$$=\left[\frac{{}^{n+1}C_0+\ {}^{n+1}C_1x+\ {}^{n+1}C_2x^2+\ {}^{n+1}C_3x^3+......+\ {}^{n+1}C_{n+1}x^{n+1}-{}^{(n+1)}C_0}{(n+1)x}\right]$$

$=\frac{[1+x)^{n+1}-1}{(n+1)x}$ = RHS

By Putting x=1 in $C_0+C_1.\frac{x}{2}+C_2.\frac{x^2}{3}+.....+C_n.\frac{x^n}{n+1}$

$=\frac{(1+x)^{n+1}-1}{(n+1)x}$ we get

$$=C_0+C_1.\frac{1}{2}+C_2.\frac{1^2}{3}+......+C_n.\frac{1^n}{n+1}=\frac{[1+1)^{n+1}-1}{(n+1)(1)}\Rightarrow C_0+\frac{C_1}{2}+\frac{C_2}{3}+.....+\frac{C_n}{n+1}=\frac{2^{n+1}-1}{n+1}$$

3) Find the sum of the series $1+\frac{4}{5}+\frac{4.7}{5.10}-\frac{4.7.10}{5.10.15}+......$

S= $1-\frac{4}{1}\left[\frac{1}{5}\right]+\frac{4.7}{1.2}\left[\frac{1}{5}\right]^2-\frac{4.7.10}{1.2.3}\left[\frac{1}{5}\right]^3+......$

Now comparing the above series with

$$1-\frac{P}{1!}\left[\frac{x}{q}\right]+\frac{P(p+q)}{2!}\left[\frac{x}{q}\right)^2-\frac{P(p+q)(p+2q)}{3!}\left[\frac{x}{q}\right)^3+.....=(1+x)^{\frac{-P}{Q}}$$

We get p=4, p+q=7 $\Rightarrow$ 4+q=7 $\Rightarrow$ q=3

Also, we have $\frac{x}{q}=\frac{1}{5}\Rightarrow x=\frac{q}{5}=\frac{3}{5}$

$$=\therefore S=(1+x)^{\frac{-p}{q}}=\left[1+\frac{3}{5}\right]^{\frac{-4}{3}}=\left[\frac{8}{5}\right]^{\frac{-4}{3}}=\left[\frac{5}{8}\right]^{\frac{4}{3}}=\frac{5^{\frac{4}{3}}}{8^{\frac{4}{3}}}=\frac{\sqrt[3]{5^4}}{(2^3)^{\frac{4}{3}}}=\frac{\sqrt[3]{625}}{(2^3)^{\frac{4}{3}}}=\frac{\sqrt[3]{625}}{2^4}=\frac{\sqrt[3]{625}}{16}$$

4) Find the sum of the infinite series $\frac{7}{5}\left[1+\frac{1}{10^2}+\frac{1.3}{1.2}.\frac{1}{10^4}+\frac{1.3.5}{1.2.3}.\frac{1}{10^6}+.....\infty\right]$

Sol: Let $S=1+\frac{1}{10^2}+\frac{1.3}{1.2}.\frac{1}{10^4}+\frac{1.3.5}{1.2.3}.\frac{10}{10^6}+.......=$

$$=1+\frac{1}{1!}\ \frac{1}{100}+\frac{1.3}{2!}\left[\frac{1}{100}\right]^2+\frac{1.3.5}{3!}\left[\frac{1}{100}\right]^3+......$$

Comparing the above series with $1+\frac{p}{1!}\left[\frac{x}{q}\right]+\frac{p(p+q)}{2!}\left[\frac{x}{q}\right]^2+.....=(1-x)^{\frac{-p}{q}}$

We get $p=1,p+q=3\Rightarrow 1+q=3\Rightarrow q=2$

Also $\frac{x}{q}=\frac{1}{100}\Rightarrow x=\frac{q}{100}=\frac{2}{100}=\frac{1}{50}$

$$\therefore S=(1-x)^{\frac{-P}{Q}}=\left[1-\frac{1}{50}\right]^{\frac{-1}{2}}=\left[\frac{49}{50}\right]^{\frac{-1}{2}}=\left[\frac{50}{49}\right]^{\frac{1}{2}}=\sqrt{\frac{50}{49}}=\frac{5\sqrt{2}}{7}$$

$\therefore$ the given series is $\frac{7}{5}(S)=\frac{7}{5}\left[\frac{5\sqrt{2}}{7}\right]=\sqrt{2}$

5) If $x=\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+......\infty$**, then find** $3x^2+6x$

Sol: $x=\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+......\infty$

Adding 1 on both sides, we have

$$\Rightarrow 1+x=1+\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+........\infty$$

$$=1+\frac{1}{1!}\left[\frac{1}{5}\right]+\frac{1.3}{2!}\left[\frac{1}{5}\right]^2+\frac{1.3.5}{3!}\left[\frac{1}{5}\right]^3+......\infty$$

Comparing the above series with $1+\frac{p}{1!}\left[\frac{y}{q}\right]+\frac{p(p+q)}{2!}\left(\frac{y}{q}\right)^2+.....=(1-y)^{\frac{-p}{q}}$

We get $p=1, p+q=3 \Rightarrow q=2$ and $\frac{y}{q}=\frac{1}{5} \Rightarrow y=\frac{q}{5}=\frac{2}{5}$

$= \therefore 1+x=(1-y)^{\frac{-P}{Q}}=\left[1-\frac{2}{5}\right)^{\frac{-1}{2}}=\left(\frac{5}{3}\right)^{\frac{1}{2}}=\sqrt{\frac{5}{3}}$

$=(1+x)^2=\frac{5}{3} \Rightarrow 1+2x+x^2=\frac{5}{3} \Rightarrow 3+6x+3x^2=5 \Rightarrow 3x^2+6x=2$

6) Find the sum of the infinite series $1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+........$

Sol: Let $S=1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+.........$ upto ∞

$=1+\frac{1}{1!}\left[\frac{1}{3}\right]+\frac{1.3}{2!}\left[\frac{1}{3}\right]^2+\frac{1.3.5}{3!}\left[\frac{1}{3}\right]^3+......$

Now comparing the above series with

$=1+\frac{p}{1!}\left[\frac{x}{p}\right]+\frac{p(p+q)}{2!}\left[\frac{x}{q}\right)^2+\frac{p(p+q)(p+2q)}{3!}\left[\frac{x}{q}\right]^3+.......$

$=(1-x)^{\frac{-p}{q}}$

We get $p=1, p+q=3 \Rightarrow 1+q=3 \Rightarrow q=2$

Also, we have $\frac{x}{q}=\frac{1}{3} \Rightarrow x=\frac{q}{3}=\frac{2}{3}$

$= \therefore S=(1-x)^{\frac{-p}{q}}=\left[1-\frac{2}{3}\right]^{\frac{-1}{2}}=\left[\frac{1}{3}\right]^{\frac{-1}{2}}=\left[\frac{3}{1}\right]^{\frac{1}{2}}=3^{\frac{1}{2}}=\sqrt{3}$

7) Prove that $C_0.C_r+C_1.C_{r+1}+C_2.C_{r+2}+....C_{n-r}.C_n={}^{2n}C_{(n+r)}$ **for** $0 \le r \le n$**. Hence deduce that**

i) $C_0^2+C_1^2+C_2^2+.....+C_n^2={}^{2n}C_n$ **ii)** $C_0C_1+C_1C_2+C_2C_3+.....+C_{n-1}.C_{n-r}={}^{2n}C_{n+r}$

Sol: **Method-I:**

We know that $(1+x)^n=C_0+C_1.x+C_2.x^2+.....+C_n.x^n........(1)$

on replacing x by $\frac{1}{x}$ in (1), we get $\left(1+\frac{1}{x}\right)^n$ = $C_0+\frac{C_1}{x}+\frac{C_2}{x^2}+......+\frac{C_n}{x_n}............(2)$

on multiplying (2) and (1)

$$\left(1+\frac{1}{x}\right)^n.(1+x)^n=\left[C_0+\frac{C_1}{x}+\frac{C_2}{x^2}+.....+\frac{C_{n-r}}{x^{n-r}}+.......+\frac{C_n}{x^n}\right]$$

$$\left[C_0+C_1x+.......+C_1x^r+C_{r+1}x^{r+1}+C_{r+2}x^{r+2}+.....+C_nx^n\right]................(3)$$

The coefficient of x^r in RHS of (3) = $C_0C_r+C_1C_{r+1}+C_2C_{r+2}+.....+C_{n-r}C_n.....(4)$

LHS of (3) is $\left(1+\frac{1}{x}\right)^n(1+x)^n=\left(\frac{(1+x)^n}{x^n}(1+x)^n\right)=\frac{(1+x)^{2n}}{x^n}$

$\therefore$ coefficient of x^r in $\frac{(1+x)^{2n}}{x^n}$ = the coefficient of x^{n+r} in $(1+x)^{2n}={}^{2n}C_{n+r}.......(5)$

Hence, from (4) and (5), we get $C_0C_r+C_1C_{r+1}+C_2C_{r+2}+....+C_{n-r}C_n={}^{2n}C_{n+r}$

i) on substituting r=0, we get $C_0^2+C_1^2+C_2^2+......+C_n^2={}^{2n}C_n$

ii) on substituting r = 1, we get $C_0C_1+C_1C_2+C_2C_3+......+C_{n-1}.C_n={}^{2n}C_{n+1}$

Method-II :

We have $(1+x)^n=C_0+C_1x+C_2x^2+......+C_rx^r+C_{r+1}x^{r+1}+C_{r+2}x^{r+2}+.....+C_nx^n.........(1)$

$\Rightarrow(x+1)^n=C_0x^n+C_1x^{n-1}+C_2x^{n-2}+.....+C_{n-r}x^{n-r}+......+C_n......(2)$

Multiplying (2) and (1), we get

$$\left(C_0x^n+C_1x^{n-1}+C_2x^{n-2}+.....+C_{n-r}x^{n-r}+C_n\right)\left(C_0+C_1x+C_2x^2+....+C_rx^r+C_{r+1}x^{r+1}+C_{r+2}x^{r+2}+....+C_nx^n\right)$$

$=(x+1)^n(1+x)^n=(1+x)^{2n}$

comparing the coefficient of x^{n+r} both sides, we get

$C_0C_r+C_1C_{r+1}+C_2C_{r+2}+......+C_{n-r}C_n={}^{2n}C_{n+r}$

8) If 36,84,126, are three successive binomial coefficients in the expansion of $(1+x)^n$, then find n.

Sol: Let the 3 successive coefficients of $(1+x)^n$ be taken as

${}^nC_{r-1}=36.....(1);\ {}^nC_r=84......(2);\ {}^nC_{r+1}=126......(3)$

Now, $\frac{(2)}{(1)}\Rightarrow\frac{{}^nC_r}{{}^nC_{r-1}}=\frac{84}{36}\Rightarrow\frac{n-r+1}{r}=\frac{1}{3}$

$\Rightarrow 3n-3r+3=7r\Rightarrow 3n-10r=-3.........(4)$

$\frac{(3)}{(2)}\Rightarrow\frac{{}^nC_{r+1}}{{}^nC_r}=\frac{126}{84}\Rightarrow\frac{n-r}{r+1}=\frac{3}{2}$

$\Rightarrow 2n\text{-}2r=3r+3 \Rightarrow 2n\text{-}5r=3........(5)$

Solving (4) and (5) we get n

$2\times(5)\Rightarrow 4n\text{-}10r=6....(6)$

Now (6) – (4) $\Rightarrow$ n=9

9) If the coefficient of x^{10} in the expansion of $\left(ax^2\text{-}\frac{1}{bx}\right)^{11}$ is equal to the coefficient of x^{-10} in the expansion of $\left(ax\text{-}\frac{1}{bx^2}\right)^{11}$. Find the relation between a and b where a and b are real numbers

Sol: In $\left(ax^2\text{-}\frac{1}{bx}\right)^{11}$, the general term

$T_{r+1}={}^{11}C_r\left[ax^2\right)^{11-r}\left[\frac{1}{bx}\right]^r={}^{11}C_r\frac{a^{11-r}}{b^r}.x^{22-3r}.........(1)$

Put $22\text{-}3r=10\Rightarrow 3r=12\Rightarrow r=4$

From (1), the coefficient of x^{10} is

${}^{11}C_4\frac{a^{11}\text{-}4}{b^4}\text{-}{}^{11}C_4\frac{a^7}{b^4}.....(2)$

In $\left[ax\text{-}\frac{1}{bx^2}\right]^{11}$ the general term is

$T_{r+1}={}^{11}C_r\left(ax\right)^{11-r}\left[\text{-}\frac{1}{bx^2}\right]^r=\left(\text{-}1\right)^r\ {}^{11}C_r\frac{a^{11}\text{-}r}{b^r}x^{11-3r}..........(3)$

Put $11\text{-}3r=\text{-}10\Rightarrow 3r=21\Rightarrow r=7$

From (3), the coefficient of x^{-10} is $(\text{-}1)^7\ {}^{11}C_7\frac{a^{11-7}}{b^7}=\text{-}{}^{11}C_7.\frac{a^4}{b^7}.........(4)$

Given that the two coefficient are equal

$\therefore$ From(2), (4), we have ${}^{11}C_4\frac{a^7}{b^4}=\text{-}{}^{11}C_7\frac{a^4}{b^7}$

$\Rightarrow\frac{a^7}{b^4}=\text{-}\frac{a^4}{b^7}\left(\because\ {}^{11}C_4={}^{11}C_7\right]$

$\Rightarrow a^3=\text{-}\frac{1}{b^3}\Rightarrow a^3b^3=\text{-}1\Rightarrow ab=\text{-}1$

10) If the coefficient of x^9, x^{10}, x^{11} in the expansion of $(1+x)^n$ are in A.P. then prove that $n^2-41n+398=0$

Sol: The coefficient of x^9, x^{10}, x^{11} in (1+x)n are

$${}^nC_9, {}^nC_{10}, {}^nC_{11}$$

Given that ${}^nC_9, {}^nC_{10}, {}^nC_{11}$ are in A.P.

$$\Rightarrow 2.{}^nC_{10} = {}^nC_9 + {}^nC_{11} \Rightarrow 2 = \frac{{}^nC_9}{{}^nC_{10}} + \frac{{}^nC_{11}}{{}^nC_{10}}$$

$$\Rightarrow 2 = \frac{10}{n-9} + \frac{n-10}{11}\left(\because \frac{{}^nC_r}{{}^nC_{r+1}} = \frac{r+1}{n-r} \,\&\, \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-r}{r+1}\right]$$

$$\Rightarrow 2 = \frac{10(11) + (n-10)(n-9)}{(n-9)(11)}$$

$$\Rightarrow 2(n-9)(11) = 110 + (n^2-19n+90)$$

$$\Rightarrow 22n-198 = n^2-19n+200$$

$$\Rightarrow n^2-41n+398=0$$

11) If the coefficient of r^{th}, $(r+1)^{th}$, $(r+2)^{th}$ terms in the expansion of $(1+x)^n$ are in A.P. then show that $n^2-(4r+1)n+4r^2-2=0$

Sol: The coefficient of r^{th}, $(r+1)^{th}$, $(r+2)^{th}$ terms in $(1+x)^n$ are ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$

Given that ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$ are in A.P.

$$\Rightarrow 2.{}^nC_r = {}^nC_{r-1} + {}^nC_{r+1} \Rightarrow 2 = \frac{{}^nC_{r-1}}{{}^nC_r} + \frac{{}^nC_{r+1}}{{}^nC_r}$$

$$\Rightarrow 2 = \frac{r}{n-r+1} + \frac{n-r}{r+1}\left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r} \text{ and } \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-r}{r+1}\right]$$

$$\Rightarrow 2 = \frac{r(r+1) + (n-r)(n-r+1)}{(n-r+1)(r+1)}$$

$$\Rightarrow 2(n-r+1)(r+1) = r(r+1) + (n-r)(n-r+1)$$

$$\Rightarrow 2nr+2n-2r^2-2r+2r+2 = r^2+r+n^2-nr+n-nr+r^2-r$$

$$\Rightarrow n^2-4nr-n+4r^2-2=0$$

$$\Rightarrow n^2-n(4r+1)+4r^2-2=0$$

12) If $x=\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+.......$ then prove that $9x^2+24x=11$

Sol: Given that $x=\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+.........$

$$=\frac{1.3}{2!}\left[\frac{1}{3}\right]^2+\frac{1.3.5}{3!}\left[\frac{1}{3}\right]^3+\frac{1.3.5.7}{4!}\left[\frac{1}{3}\right]^4+..........$$

Adding $1+\frac{1}{3}$ on both sides, we have

$$1+\frac{1}{3}+x=1+\frac{1}{1!}\left[\frac{1}{3}\right]+\frac{1.3}{2!}\left[\frac{1}{3}\right]^2+\frac{1.3.5}{3!}\left[\frac{1}{3}\right]^3+.....$$

Comparing the above series with

$$1+\frac{p}{1!}\left[\frac{y}{p}\right]+\frac{p(p+q)}{2!}\left[\frac{y}{q}\right]^2+.......=[1-y]^{\frac{-p}{q}}$$

we get, $p=1, p+q=3 \Rightarrow 1+q=3 \Rightarrow q=2$

Also $\frac{y}{p}=\frac{1}{3} \Rightarrow y=\frac{q}{3}=\frac{2}{3}$

$$\therefore 1+\frac{1}{3}+x=(1-y)^{\frac{-p}{q}}=\left[1-\frac{2}{3}\right]^{\frac{-1}{2}}=\left[\frac{1}{3}\right]^{\frac{-1}{2}}=(3)^{\frac{1}{2}}=\sqrt{3}$$

$$=\frac{4}{3}+x=\sqrt{3} \Rightarrow x=\sqrt{3}-\frac{4}{3}=\frac{3\sqrt{3}-4}{3}=\frac{3\sqrt{3}-4}{3}$$

$$\Rightarrow 3x=3\sqrt{3}-4 \Rightarrow 3x+4=3\sqrt{3}$$

$$\Rightarrow (3x+4)^2=\left(3\sqrt{3}\right)^2 \Rightarrow 9x^2+24x+16=27$$

$$\Rightarrow 9x^2+24x=11$$

Very Short Answer Questions **(2 Marks)**

1) Find the middle term(s) in the expansion of $\left(4x^2+5x^3\right)^{17}$

Sol: Given binomial exponent n=17 is odd

$\therefore$ 2 middle terms are $\frac{T_{17+1}}{2}=\frac{T_{18}}{2}=T_9$ and next term T_{10} in $(4x^2+5x^3)^{17}$ we have

$$T_9=T_{8+1}={}^{17}C_8\left(4x^2\right)^{17-8}\left(5x^3\right)^8={}^{17}C_8 4^9\left(x^2\right)^9 5^8\left(x^3\right)^8={}^{17}C_8.4^9.5^8.X^{42}$$

also $T_{10}=T_{9+1}={}^{17}C_9\left(4x^2\right)^{17-9}\left(5x^3\right)^9={}^{17}C_9 4^8\left(x^2\right)^8.5^9.\left(x^3\right)^9={}^{17}C_9 4^8.5^9 x^{43}$

2) Find the coefficient of x^{-6} in $\left[3x-\frac{4}{x}\right]^{10}$

Sol: General form of $\left(3x-\frac{4}{x}\right)^{10}$ is $T_{r+1}={}^{10}C_r\left(3x\right)^{10-r}\left(\frac{-y}{x}\right)^{r}$

$=\left(-1\right)^{r}\ {}^{10}C_1 3^{10-r}4^{r}x^{10-2r}........(1)$

To get the coefficient of x^{-6} put $10-2r=-6\Rightarrow 2r=10\Rightarrow r=8$

From (1), the coefficient of x^{-6} is $(-1)^{8}\ {}^{10}C_8 3^{10-8}4^{8}={}^{10}C_8 3^{2}4^{8}$

3) If the coefficients of $\left(2r+4\right)^{th}.\left(r-2\right)^{th}$ terms in the expansion of $(1+x)^{18}$ are equal. Find r.

Sol: We know coefficient of r^{th} term in $(1+x)^n$ is ${}^{n}C_{r-1}$

Given that in $(1+x)^{18}$, the coefficient of $(2r+4)^{th}$ term = coefficient of $(r-2)^{th}$ term

$\Rightarrow {}^{18}C_{2r+3}={}^{18}C_{r-3}$

$\therefore 2r+3=r-3 \text{ or } \left(2r+3\right)+\left(r-3\right)=18 \qquad \left[\because\ {}^{n}C_r={}^{n}C_s\Rightarrow r=s\ (or)\ r+s=n\right]$

$\Rightarrow r=-6$ But r cannot be negative (or) $3r=18\Rightarrow r=6$

4) Find the middle term (S) in $\left[4a+\frac{3b}{2}\right]^{11}$

Sol: The binomial exponent n=11 is odd

$\therefore$ The 2 middle terms are $\frac{T_{11+1}}{2}=\frac{T_{12}}{2}=T_6$ and the next form T_7 in

$\left[4a+\frac{3b}{a}\right]^{11} T_6=T_{5+1}={}^{11}C_5\left(4a\right)^{6}\left(\frac{3}{2}b\right)^{5}={}^{11}C_5 4^{6}.\frac{3^5}{2^5}a^{6}b^{5}=77\times 2^{8}\times 3^{6}\times a^{6}b^{5}$

$T_7=T_{6+1}={}^{11}C_6\left(4a\right)^{5}\left(\frac{3}{2}b\right)^{6}={}^{11}C_5 4^{5}\frac{3^6}{2^6}a^{5}b^{6}=77\times 2^{5}\times 3^{7}\times a^{5}b^{6}$

5) Fin the term independent of x in $\left[\frac{3}{\sqrt[3]{x}}+5\sqrt{x}\right]^{25}$

Sol: General term of $\left[\frac{3}{\sqrt[3]{x}}+5\sqrt{x}\right]^{25}$ is

$T_{r+1}={}^{25}C_r\left[\frac{3}{\sqrt[3]{x}}\right]^{25-r}\left[5\sqrt{x}\right]^{r}={}^{25}C_r 3^{25-r}5^{r}x^{\frac{-25+r}{3}+\frac{r}{2}}$

To get the term independent of x, we put

$$\frac{-25+r}{3}+\frac{r}{2}=0 \Rightarrow \frac{25-r}{3}=\frac{r}{2} \Rightarrow 50-2r=3r \Rightarrow 5r=50 \Rightarrow r=10$$

$\therefore$ From (1), the term independent of x is 25.

$$C_{10}3^{25-10}5^{10}={}^{25}C_{10}3^{15}5^{10}$$

6) Write the general term in $\left[1-\frac{5x}{3}\right]^{-3}$

Sol: General term of $(1-x)^{-n}$ is $T_{r+1}=\frac{n(n+1).......(n+r-1)}{r!}x^r$ Hence n = 3, $x=\frac{5x}{3}$

$$\therefore T_{r+1}=\frac{(3)(3+1)(3+2).......(3+r-1)}{r!}\left[\frac{5x}{3}\right]^r=\frac{(3)(4)(5)........(r+2)}{r!}\left[\frac{5x}{3}\right]^r$$

7) Find the general (r+1)th term in the expansion of $(4+5x)^{\frac{-3}{2}}$

Sol: G.E. = $(4+5x)^{\frac{-3}{2}}=\left[4\left(1+\frac{5}{4}x\right)\right]^{\frac{-3}{2}}=\left(2^r\right)^{\frac{-3}{2}}\left[\left(1+\frac{5}{4}x\right)^{\frac{-3}{2}}\right]=\frac{1}{8}$

$=\left[\left(1+\frac{5x}{4}\right)^{\frac{-3}{2}}\right]$ Here $n=\frac{3}{2},x=\frac{5x}{4}$

General term of $(1+x)^{-n}$ is $T_{r+1}=(-1)^r\frac{n(n+1)(n+r-1)}{r!}x^r$

$$\therefore T_{r+1}=\frac{1}{8}\left[\frac{\left(\frac{3}{2}\right)\left(\frac{3}{2}+1\right)\left(\frac{3}{2}+2\right)......\left(\frac{3}{2}+r-1\right)}{r!}\right]\left(\frac{5x}{4}\right)^r$$

$$\therefore T_{r+1}=\frac{1}{8}\left[\frac{(3)(5)(7)........\left(2^{r+1}\right)}{2^r(r!)}\right]\left[\frac{5x}{4}\right]^r$$

Long Answer Questions **(7 Marks)**

1) If the 2nd , 3rd and 4th terms in the expansion of (a+x)^n are respectively 24,720 and 1080. Then find the value of a, x and n

Sol: The second term of (a+x)^n is $T_r=T_{r+1}={}^nC_1a^{n-1}x^{-1}=240....(1)$

The third term of (a+x)^n is $T_3=T_{2+1}={}^nC_2a^{n-2}x^2=720......(2)$

The fourth term of (a+x)^n is $T_4=T_{3+1}={}^nC_3a^{n-3}x^3=1080.......(3)$

$$\frac{(2)}{(1)} \Rightarrow \frac{{}^nC_2a^{n-2}x^2}{{}^nC_1a^{n-1}x} = \frac{720}{240} \Rightarrow \left[\frac{{}^nC_r}{{}^nC_1}\right](a^{-1})(x)=3 \Rightarrow \left[\frac{n-1}{2}\right]\left[\frac{x}{a}\right]=3$$

$$\Rightarrow (n-1)(x)=6a.......(4) \qquad \left[\because \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-r}{r+1}\right]$$

$$\frac{(3)}{(2)} \Rightarrow \frac{{}^nC_3a^{n-3}x^3}{{}^nc_2a^{n-2}x^2} = \frac{1080}{720} = \frac{9}{6} = \frac{3}{2} \Rightarrow \left[\frac{{}^nC_3}{{}^nC_r}\right](a^{-1})(x) = \frac{3}{2} \Rightarrow \left[\frac{n-2}{3}\right]\left[\frac{x}{a}\right] = \frac{3}{2}$$

$$\Rightarrow 2(n-2)(x)=9a.......(5)$$

$$\frac{(5)}{(4)} \Rightarrow \frac{2(n-2)(x)}{(n-1)(x)} = \frac{9a}{6a} = \frac{3}{2} \Rightarrow 4(n-2)=3(n-1) \Rightarrow 4n-8=3n-3 \Rightarrow n=5$$

Now (4) $\Rightarrow (5-1)x=6a \Rightarrow 4x=6a \Rightarrow 2x=3a \Rightarrow x=\frac{3a}{2}..........(6)$

Also (1) $\Rightarrow {}^nC_1a^{n-1}\alpha^1=240 \Rightarrow {}^na_{n-1}x=240$

$$\Rightarrow 5a^{5-1}\left[\frac{3a}{2}\right]=(24)(10) \Rightarrow a^4a=\frac{(24)(10)(2)}{(5)(3)}=32 \Rightarrow a^5=32=2^5 \Rightarrow a=2$$

$$\therefore \text{From (1) } x=\frac{3a}{2}=\frac{3(r)}{2}=3 \qquad \therefore a=2,x=3,n=5$$

2) Prove that (i) $C_0 + 3C_1+3^2.C_2+........+3^nC_n=4^n$ ii) $\frac{C_1}{C_0}+2.\frac{C_2}{C_1}+3.\frac{C_3}{C_2}+....+n.\frac{C_n}{C_{n-1}}=\frac{n(n+1)}{2}$

Sol: (i) we have $(1+x)^n = C_0+C_1x+C_2x^2+.....+C_nx^n$

Put x=3, we get $C_0+C_1.3+C_2.3^2+......+C_n.3^n=(1+3)^n=4^n$

(ii) $\frac{C_1}{C_0}+2.\frac{C_2}{C_1}+3.\frac{C_3}{C_2}+......+n.\frac{C_n}{C_{n-1}} = \frac{{}^nC_1}{{}^nC_0}+2.\left[\frac{{}^nC_r}{{}^nC_1}\right]+3.\left[\frac{{}^nC_3}{{}^nC_2}\right]+....+n.\left[\frac{{}^nC_n}{{}^nC_{n-1}}\right]$

$=\frac{n}{1}+2.\frac{(n-1)}{2}+3.\frac{(n-2)}{3}+.....+n\frac{(1)}{n}$

$=n+(n-1)+(n-2)+....+3+2+1$

$=1+2+3+.....+(n-1)+n=\frac{(n)(n+1)}{2}$

3) Find the sum of infinite series $1+\frac{2}{3}.\frac{1}{2}+\frac{2.5}{3.6}.\left[\frac{1}{2}\right]^2+\frac{2.5.8}{3.6.9}\left[\frac{1}{2}\right]^3+......\infty$

Sol: Let $S=1+\frac{2}{3}.\frac{1}{2}+\frac{2.5}{3.6}\left[\frac{1}{2}\right]^2+\frac{2.5.8}{3.6.9}\left[\frac{1}{2}\right]^3+.....$

$=1+\frac{2}{1}.\frac{1}{6}+\frac{2.5}{1.2}\left[\frac{1}{6}\right]^2+\frac{2.5.8}{1.2.3}\left[\frac{1}{6}\right]^3+......$

Comparing the above series with

$1+\frac{p}{1!}\left[\frac{x}{q}\right]+\frac{p(p+q)}{2!}\left[\frac{x}{q}\right]^2+\frac{p(p+q)(p+2q)}{3!}\left[\frac{x}{q}\right]^3+.........$

$=(1-x)^{-p/q}$

we get $p=2, p+q=5 \Rightarrow 2+q=5 \Rightarrow q=3$ also we have

$\frac{x}{q}=\frac{1}{6} \Rightarrow x=\frac{q}{6}=\frac{3}{6}=\frac{1}{2}$

$\therefore S=(1-x)^{\frac{-p}{q}}=\left[1-\frac{1}{2}\right]^{\frac{-2}{3}}=\left[\frac{1}{2}\right]^{\frac{-2}{3}}=\left[\frac{2}{1}\right]^{\frac{2}{3}}=2^{\frac{2}{3}}$

$=\left[2^2\right]^{\frac{1}{3}}=4^{\frac{1}{3}}=\sqrt[3]{4}$

4) Find the sum of the infinite series $\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+.........$

Sol: Let $S=\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+.......=\frac{3}{1}.\frac{1}{4}+\frac{3.5}{1.2}\left[\frac{1}{4}\right]^2+\frac{3.5.7}{1.2.3}\left[\frac{1}{4}\right]^3+.........$

$\Rightarrow 1+S=1+\frac{3}{1}.\frac{1}{4}+\frac{3.5}{1.2}\left[\frac{1}{4}\right]^2+..........$

comparing the above series with $1+\frac{p}{1!}\left[\frac{x}{q}\right]+\frac{p(p+q)}{2!}\left[\frac{x}{q}\right]+............=(1-x)^{\frac{-p}{q}}$

we get $p=3, p+q=5 \Rightarrow 3+q=5 \Rightarrow q=2$ Also $\frac{x}{q}=\frac{1}{4}$

$\Rightarrow x=\frac{q}{4}=\frac{2}{4}=\frac{1}{2}$

$\therefore 1+S=(1-x)^{\frac{-p}{q}}=\left[1-\frac{1}{2}\right]^{\frac{-3}{2}}=\left[\frac{1}{2}\right]^{\frac{-3}{r}}=2^{\frac{3}{r}}$

$=\left(2^3\right)^{\frac{1}{2}}=8^{\frac{1}{2}}=\sqrt{8}=2\sqrt{2}$

Hence $S=2\sqrt{2}-1$

5) Find the sum of the infinite series $\frac{3.5}{5.10}+\frac{3.5.7}{5.10.15}+\frac{3.5.7.9}{5.10.15.20}+.........\infty$

Sol: Let $S=\frac{3.5}{5.10}+\frac{3.5.7}{5.10.15}+\frac{3.5.7.9}{5.10.15.20}+..........\infty=1+\frac{3}{1}\left[\frac{1}{5}\right]+\frac{3.5}{1.2}\left[\frac{1}{5}\right]^2+.....$

Adding $1+3.\frac{1}{5}$ on both sides, we have

$1+3.\frac{1}{5}+5=1+\frac{3}{1}.\left[\frac{1}{5}\right]+\frac{3.5}{1.2}\left[\frac{1}{5}\right]^2+.........$

comparing the above series with $1+\frac{p}{1}\left[\frac{x}{q}\right]+\frac{p(p+q)}{1.2}\left[\frac{x}{q}\right]^2+.......=(1-x)\frac{-p}{q}$

we get $p=3\Rightarrow p+q=5\Rightarrow 3+q=5\Rightarrow q=2$ Also $\frac{x}{q}=\frac{1}{5}\Rightarrow x=\frac{q}{5}=\frac{2}{5}$

$\therefore 1+\frac{3}{5}+5=(1-x)^{\frac{-p}{Q}}=\left[1-\frac{2}{5}\right]^{\frac{-3}{r}}=\left[\frac{3}{5}\right]^{\frac{-3}{r}}=\left[\frac{5}{3}\right]^{\frac{3}{r}}$

$=\frac{5\sqrt{5}}{3\sqrt{3}}\Rightarrow\frac{8}{5}+5=\frac{5\sqrt{5}}{3\sqrt{3}}\Rightarrow 5=\frac{5\sqrt{5}}{3\sqrt{3}}=-\frac{8}{5}$

6) **If** $t=\frac{4}{5}+\frac{4.6}{5.10}+\frac{4.6.8}{5.10.15}+...........$ **then prove that 9t = 16**

Sol: Given that $t=\frac{4}{5}+\frac{4.6}{5.10}+\frac{4.6.8}{5.10.15}+...........$

Adding 1 on both sides, we have

$1+t=1+\frac{4}{1!}\left[\frac{1}{5}\right]+\frac{4.6}{2!}\left[\frac{1}{5}\right]^2+\frac{4.6.8}{3!}\left[\frac{1}{5}\right]^3+..........$

Comparing the above series with $1+\frac{p}{1!}\left[\frac{x}{q}\right]+\frac{p(p+q)}{2!}\left[\frac{x}{q}\right]^2+.............=(1-x)^{\frac{-p}{q}}$

We get $p=4,\ p+q=6\Rightarrow 4+q=6\Rightarrow q=2$ Also $\frac{x}{q}=\frac{1}{5}\Rightarrow x=\frac{q}{5}=\frac{2}{5}$

$\therefore 1+t=(1-x)^{\frac{-p}{q}}=\left(1-\frac{2}{5}\right)^{\frac{-4}{2}}=\left[\frac{3}{5}\right]^{-2}=\left[\frac{5}{3}\right]^2=\frac{25}{9}$

$\Rightarrow 1+t=\frac{25}{9}\Rightarrow 9(1+t)=25\Rightarrow 9+9t=25\Rightarrow 9t=16$

7) **If** $x=\frac{5}{2!3}+\frac{5.7}{3!3^2}+\frac{5.7.9}{4!.3^3}+.......\infty$ **, then find the value of** x^2+4x

Sol: Given that $x=\frac{5}{2!3}+\frac{5.7}{3!.3^2}+\frac{5.7.9}{4!3^3}+..........\infty=\frac{3.5}{2!3^2}+\frac{3.5.7}{3!.3^3}+\frac{3.5.7.9}{4!3^4}$

$\frac{3.5}{2!}\left[\frac{1}{3}\right]^2+\frac{3.5.7}{3!}\left[\frac{1}{3}\right]^3+\frac{3.5.7.9}{4!}\left[\frac{1}{3}\right]^4+...........$

Adding $1+\frac{3}{1}\left[\frac{1}{3}\right]$ on both sides, we have

Now, $1+\frac{3}{1}\left[\frac{1}{3}\right]+x=1+\frac{3}{1}\left[\frac{1}{3}\right]+\frac{3.5}{2!}\left[\frac{1}{3}\right]^2+\frac{3.5.7}{3!}\left[\frac{1}{3}\right]^3+\frac{3.5.7.9}{4!}\left[\frac{1}{3}\right]^4+......$

Comparing the above series with $1+\frac{p}{1!}\left[\frac{y}{q}\right]+\frac{p(p+q)}{2!}\left[\frac{y}{q}\right]^2+.......=(1-y)^{\frac{-p}{q}}$

We get $p=3,p+q=5\Rightarrow 3+q=5\Rightarrow q=2$ Also $\frac{y}{q}=\frac{1}{3}\Rightarrow y=\frac{q}{3}=\frac{2}{3}$

$\therefore 1+\frac{3}{1}\left[\frac{1}{3}\right]+x=(1-y)^{\frac{-p}{q}}=\left[1-\frac{2}{3}\right]^{\frac{-3}{2}}=\left[\frac{-1}{3}\right]^{\frac{-3}{2}}=(3)^{\frac{3}{2}}=(3^3)^{\frac{1}{2}}=\sqrt{27}$

$\Rightarrow 1+1+x=\sqrt{27}\Rightarrow 2+x=\sqrt{27}\Rightarrow (2+x)^2=27$

$\Rightarrow x^2+4x+4=27\Rightarrow x^2+4x-23$

8) If R, n are positive integers n is odd 0<F<1 and if $(5\sqrt{5}+11)^n=R+F$, then prove that (i) R is an even integers and (ii) (R+F), F=4^n.

Sol: Given that $(5\sqrt{5}+11)^n=R+F$; Let $G=(5\sqrt{5}-11)^n\Rightarrow 0<G<1$

Now $(R+F)-G=(5\sqrt{5}+11)^n-(5\sqrt{5}-11)^n$

$=\left[{}^nC_0(5\sqrt{5})^n+{}^nC_1(5\sqrt{5})^{n-1}(11)+{}^nC_r(5\sqrt{5})^{n-1}(11)^2+......+{}^nC_n(11)^n\right]$

$-\left[{}^nC_0(5\sqrt{5})^n-{}^nC_1(5\sqrt{5})^{n-1}(11)+{}^nC_2(5\sqrt{5})^{n-2}(11)^2+......+{}^nC_n(-11)^3\right]$

$=2\left[{}^nC_1(5\sqrt{5})^{n-1}(11)+{}^nC_3(5\sqrt{5})^{n-1}(11)^2+.....\right]$ = 2(an integer) = An Even integer

$\therefore$ R+F-G is an Even integer $\Rightarrow$ F-G is an integer since R is an integer

But 0<F<1 and -1<-G<0 $\Rightarrow$ 1<F-G<1 $\Rightarrow$ F-G=0 $\Rightarrow$ F=G $\therefore$ R is an Even integer

(ii) $(R+F)F=(R+F)G=(5\sqrt{5}+11)^n(5\sqrt{5}-11)^n=\left[(5\sqrt{5}+11)(5\sqrt{5}-11)\right]^n-(125-121)^n=4^n$

CHAPTER: 7

PARTIAL FRACTIONS

Weightage: (4 M)

KEY CONCEPTS

- **Type-1:** It's in the form $\frac{f(x)}{g(x)}$ where g(x) contains non-repeated linear factors in the form ax + b.

  Here, for every factor (ax + b) there exists one partial fraction of the $\frac{A}{ax+b}$

  $$\textbf{Ex :}\ \frac{2x+3}{(x+2)(2x+1)}=\frac{A}{x+2}+\frac{B}{2x+1}$$

- **Type-2 :** It's in the form $\frac{f(x)}{g(x)}$ where g(x) contains repeated and non-repeated linear factors in the form $(ax + b)^n$.

  Here, for every repeated factor $(ax + b)^n$, $n>1\in N$, there exists n partial fractions of the form

  $$\frac{A_1}{ax+b}+\frac{A_2}{(ax+b)^2}+........+\frac{A_n}{(ax+b)^n}$$

  $$\textbf{Ex:}\ \frac{x^2+13x+15}{(2x+3)(x+3)^2}=\frac{A}{2x+3}+\frac{B}{x+3}+\frac{C}{(x+3)^2}$$

- **Type-3 :** It's in the form $\frac{f(x)}{g(x)}$, where g(x) contains a repeated irreducible factor of the form $(ax^2+bx+c)^2$

  Here, for every factor $(ax^2+bx+c)^2$ there exists partial fractions of the form $\frac{Ax+B}{ax^2+bx+c}+\frac{Cx+D}{(ax^2+bx+c)^2}$

Ex: $\frac{x^2+1}{(x^2+x+1)^2}=\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{(x^2+x+1)^2}$

➢ **Type-4 :** Its an improper rational function of the form $\frac{f(x)}{g(x)}$ where g(x) contains linear factors or repeated linear factors

Here, first express the improper rational function

$\frac{f(x)}{g(x)}$ as $\frac{f(x)}{g(x)}=q(x)+\frac{r(x)}{g(x)}$ and resolve $\frac{r(x)}{g(x)}$ into its partial fractions accordingly

Ex: $\frac{x^3}{(x-1)(x+2)}-(x-1)+\frac{3x-2}{x^2+x-2}=(x-1)+\frac{A}{x-1}+\frac{B}{x+2}$

➢ **Type-5 :** It's in the form $\frac{f(x)}{g(x)}$ where g(x) single repeated linear factor in the form $(ax + b)^n$

Here take g(x) = y and find x in terms of y

Then change $\frac{f(x)}{g(x)}$ into a rational function of y and simplify accordingly

Ex: $\frac{x^2-2x+6}{(x-2)^3}=\frac{(y+2)^2-2(y+2)+6}{y^3}$ where y = x - 2

➢ **Type 6 :** It's in the form $\frac{f(x)}{g(x)}$, where g(x) contains a non repeated irreducible factor of the form ax^2+bx+c

Here, for every factor (ax^2+bx+c), there exists one partial fraction of the form $\frac{Ax+B}{ax^2+bx+c}$

Ex: $\frac{x^2-3}{(x+2)(x^2+1)}=\frac{A}{x+2}+\frac{Bx+C}{x^2+1}$

LEVEL-I (4 Marks)

1) Resolve $\frac{1}{x^3(x+a)}$ into partial fractions

Sol: Let $\frac{1}{x^3(x+a)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{D}{x+a}$

$$= \frac{Ax^2(x+a)+Bx(x+a)+C(x+a)+Dx^3}{x^3(x+a)}$$

$$= Ax^2(x+a)+Bx(x+a)+C(x+a)+Dx^3=1..........(1)$$

Putting x = -a in (1) we get $A(0)+B(0)+C(0)+D(-a)^3=1$

$$= D(-a)^3 = 1 \Rightarrow D=\frac{-1}{a^3}$$

Putting x = -0 in (1) we get A.0 (a) + B.0(a) +C(a)+D(0)=1

$$\Rightarrow C(a) = 1 \Rightarrow C = \frac{1}{a}$$

Equating the coefficient of x^3, we get

$$A+D=0 \Rightarrow A=-D \Rightarrow A=\frac{1}{a^3}$$

Equating the coefficient of x^2, we get

$$a(A)+B=0 \Rightarrow B=-a(A)=-a\left[\frac{1}{a^3}\right] \Rightarrow B=\frac{-1}{a^2}$$

$$\therefore \frac{1}{x^3(x+a)}=\frac{A}{x}+\frac{B}{x^2}-\frac{C}{x^3}+\frac{D}{x+a}$$

$$\frac{1}{a^3x}-\frac{1}{a^2x^2}+\frac{1}{ax^3}-\frac{1}{a^3(x+a)}$$

2) Resolve $\frac{2x+3}{(x-1)^3}$ into partial fractions

Sol: x-1=y then x=y+1 $\therefore \frac{2x+3}{(x-1)^3}=\frac{2(y+1)+3}{y^3}=\frac{2y+5}{y^3}$

$$\frac{2}{y^2}+\frac{5}{y^3}=\frac{2}{(x-1)^2}+\frac{5}{(x-1)^3}$$

$$\therefore \frac{2x+3}{(x-1)^3}=\frac{2}{(x-1)^2}+\frac{5}{(x-1)^3}$$

3) Resolve $\frac{3x-1}{(1-x+x^2)(x+2)}$ into partial fractions

Sol: Let $\frac{3x-1}{(1-x+x^2)(x+2)}=\frac{A}{x+2}+\frac{Bx+C}{1-x+x^2}=\frac{A(1-x+x^2)+(Bx+C)(x+2)}{(x+2)(1-x+x^2)}$

$\Rightarrow A(1-x+x^2)+(Bx+C)(x+2)=3x-1$...............(1)

Putting x=-2 in (1) we get A(1+2+4)=-7

$\Rightarrow$7A=-7 $\Rightarrow$ A=-1

Equating the coefficients of x^2 in (1) we get A+B=0

Equating the constant terms in (1), we get A+2C=-1

$\Rightarrow$ B = - A

$\Rightarrow$2C=-1-A=-1+1=0$\Rightarrow$C=0

$$\therefore \frac{3x-1}{\left(1-x+x^2\right)\left(x+2\right)}=\frac{A}{x+2}+\frac{Bx+C}{1-x+x^2}=\frac{-1}{x+2}+\frac{x}{1-x+x^2}$$

4) Resolve $\frac{2x^2+2x+1}{x^3+x^2}$ into partial fractions

Sol: The denominator $x^3+x^2=x^2(x+1)$

G.E. = $\frac{2x^2+2x+1}{x^3+x^2}=\frac{2x^2+2x+1}{x^2(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}=\frac{Ax\left(x+1\right)+B\left(x+1\right)+Cx^2}{x^2\left(x+1\right)}$

$\Rightarrow$Ax(x+1)+B(x+1)+Cx^2=2x^2+2x+1………… (1)

Putting x=0 in(1) we get A(0)+B(1)+C(0)=1$\Rightarrow$B=1

Putting x=-1 in (1) we get A(0)+B(0)+C$(-1)^2$

= 2$(-1)^2$+2(-1)+1$\Rightarrow$C(1)=1$\Rightarrow$C=1

Equating the coefficients of x^2, we get 2=A+C$\Rightarrow$A=2-C

=2-1=1

$$\therefore \frac{2x^2+2x+1}{x^3+x^2}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}=\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x+1}$$

5) Resolve $\frac{x^2-3}{\left(x+2\right)\left(x^2+1\right)}$ into partial fractions

Sol: Let $\frac{x^2-3}{\left(x+2\right)\left(x^2+1\right)}=\frac{A}{x+2}+\frac{Bx+C}{x^2+1}=\frac{A\left(x^2+1\right)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$

$\Rightarrow$A(x^2+1)+(Bx+C)(x+2)=x^2-3………….(1)

Putting x=-2 in (1) we get A(4+1)+(Bx+C)(0)=4-3

= 5A=1$\Rightarrow$A=$\frac{1}{5}$

Putting x=0 in (1), we get A+2C=-3$\Rightarrow$C=$\frac{-8}{5}$

Comparing the coefficients of x^2, we get A+B=1

$\Rightarrow$ B=1-A=$1-\frac{1}{5}=\frac{4}{5}$

$$\therefore \frac{x^2-3}{(x+2)(x^2+1)}=\frac{1}{5(x+2)}+\frac{4x-8}{5(x^2+1)}$$

6) Resolve $\frac{x^2-x+1}{(x+1)(x-1)^2}$ into partial fractions

Sol: Let $\frac{x^2-x+1}{(x+1)(x-1)^2}=\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$

$$\Rightarrow \frac{x^2-x+1}{(x+1)(x-1)^2}=\frac{A(x-1)^2+B(x+1)(x-1)+C(x+1)}{(x+1)(x-1)^2}$$

$\Rightarrow$ A(x-1)^2+B(x+1)(x-1)+C(x+1)=x^2-x+1.....(1)

Putting x=1 in (1) we get A(1-1)^2+B(2)(0)+C(1+1)=1

$\Rightarrow$ 2C=1 $\Rightarrow$ C=$\frac{1}{2}$

Putting x=-1 is (1), we get A(-1-1)^2+B(-1+1)(-1-1)+C(-1+1)=3

$\Rightarrow$ 4A=3 $\Rightarrow$ A=$\frac{3}{4}$

Equating the coefficients of x^2, we get A+B=1 $\Rightarrow$ B=1-A=$1-\frac{3}{4}=\frac{1}{4}$

$$\therefore \frac{x^2-x+1}{(x+1)(x-1)^2}=\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}=\frac{3}{4(x+1)}+\frac{1}{4(x-1)}+\frac{1}{2(x-1)^2}$$

7) Resolve $\frac{2x^2+3x+4}{(x-1)(x^2+2)}$ into partial fractions

Sol: Let $\frac{2x^2+3x+4}{(x-1)(x^2+2)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+2}=\frac{A(x^2+2)+(Bx+C)(x-1)}{(x-1)(x^2+2)}$

$\Rightarrow$ A(x^2+2)+(Bx+C)(x-1)=2x^2+3x+4......(1)

Putting x=1 in (1) we get A(1^2+2)+(Bx+C)(0)=2(1)^2+3(1)+4

$\Rightarrow$ 3A=9 $\Rightarrow$ A=3

Putting x=0 in(1), we get A(0+2)+(0+C)(0-1)=4 $\Rightarrow$ 2A-C=4

$\Rightarrow$ C=2A-4=2(3)-4=2

Comparing the coefficients of x^2 in(1), we get A+B=2$\Rightarrow$0

$\Rightarrow$3+B=2$\Rightarrow$B=-1

$$\therefore \frac{2x^2+3x+4}{(x-1)(x^2+2)}=\frac{3}{x-1}+\frac{(-1)x+2}{x^2+2}+\frac{3}{x-1}+\frac{2-x}{x^2+2}$$

8) Resolve $\frac{x^4}{(x-1)(x-2)}$ into partial fractions

Sol: (x-1(x-2)=x^2-3x+2. Now on dividing x^4 by x^2-3x+2,

$$x^2-3x+2)\,x^4\,(x^2+3x+7$$
$$x^4 - 3x^3 + 2x^2$$
$$(-)\ (+)\ (-)$$
$$3x^3 - 2x^2$$
$$3x^3 - 9x^2 + 6x$$
$$(-)\ (+)\ (-)$$
$$7x^2-6x$$
$$7x^2 - 21x + 14$$
$$(-)\ (+)\ (-)$$
$$15x-14$$

$$\frac{x^4}{(x-1)(x-2)}$$

$$= (x^2+3x+7)+\frac{15x-14}{(x-1)(x-2)}$$

$$\frac{15x-14}{(x-1)(x-2)} = \frac{A}{(x-1)}+\frac{B}{(x-2)}=\frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$$

A(x-2)+B(x-1)=15x-14……(1)

Putting x=1 in (1), we get A(1-2)+B(0)=15(1)-14=1$\Rightarrow$A=-1

Putting x=2 in (1), we get A(0)+B(2-1)=15(2)-14=16$\Rightarrow$B=16

$$\therefore \frac{x^4}{(x-1)(x-2)}=x^2+3x+7-\frac{1}{x-1}+\frac{16}{x-2}$$

9) Resolve $\frac{3x-18}{x^3(x+3)}$ into partial fractions

Sol: Let $\frac{3x-18}{x^3(x+3)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{D}{x+3}\Rightarrow\frac{3x+8}{x^3(x+3)}$

$$= \frac{Ax^2(x+3)+Bx(x+3)+C(x+3)+Dx^3}{x^3(x+3)}$$

$$\Rightarrow Ax^2(x+3)+Bx(x+3)+C(x+3)+Dx^3=3x-18……(1)$$

Putting x=-3 in (1), we get A(0)+B(0)+C(0)+D$(-3)^2$

= $3(-3)^2$-18$\Rightarrow$-270=-27$\Rightarrow$D=1

Putting x=0 in (1) we get A(0)+B(0)+C(0+3)+D(0)=3(0)-18

$\Rightarrow$3C=-18$\Rightarrow$C=-6

Equating the coefficients of x^3 in(1), we get

$\Rightarrow$A+D=0$\Rightarrow$A=-D$\Rightarrow$-1$\Rightarrow$A=-1

Equating the coefficients of x^2 in (1), we get

$\Rightarrow$3A+B=0$\Rightarrow$ B=-3A=-3(-1)=3$\Rightarrow$B=3

$$\therefore \frac{3x-18}{x^3(x+3)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{D}{x+3}$$

$$= \frac{-1}{x}+\frac{3}{x^2}-\frac{6}{x^3}+\frac{1}{x+3}$$

10) Resolve $\frac{x^3}{(x-1)(x+2)}$ into partial fractions

Sol: Here, then degree of numerator of $3\geq$ degree of denominator 2.

So, it is an improper function

$$x^2+x-2\overline{)\,x^3\,}(x-1$$
$$x^3+x^2-2x$$
$$(-)\ (-)\ (+)$$
$$-x^2+2x$$
$$-x^2-x+2$$
$$(+)\ (+)\ (-)$$
$$3x-2$$

Also (x-1)(x+2)=x^2+x-2

$$\frac{x^3}{(x-1)(x+2)}=(x-1)+\frac{3x-2}{x^2+x-2}$$

$$\frac{3x-2}{x^2+x-2}=\frac{3x-2}{(x-1)(x+2)}$$

$$= \frac{A}{x-1}+\frac{B}{x+2}=\frac{A(x+2)+B(x-1)}{(x-1)(x+2)}$$

$\Rightarrow$A(x+2)+B(x-1)=3x-2……(1)

Putting x=1 in (1), we get A(3)+B(0)=3-2$\Rightarrow$3A=1$\Rightarrow$A=$\frac{1}{3}$

Putting x=-2 in (1), we get A(0)+B(-3)=3(-2)(-2)

$$\Rightarrow -3B=-8\Rightarrow B=\frac{8}{3}$$

$$\Rightarrow \frac{3x-2}{x^2+x-2}=\frac{A}{x-1}+\frac{B}{x+2}=\frac{1}{3(x-1)}+\frac{8}{3(x+1)}$$

$$\therefore \frac{x^3}{(x-1)(x+2)}=x-1+\frac{1}{3(x-1)}+\frac{8}{3(x+1)}$$

LEVEL-II (4 Marks)

1) Resolve $\frac{x^2+1}{(x^2+x+1)^2}$ into partial fractions

Sol: Let $\frac{x^2+1}{(x^2+x+1)^2}=\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{(x^2+x+1)^2}=\frac{(Ax+B)(x^2+x+1)+Cx+D}{(x^2+x+1)^2}$

$\Rightarrow$(Ax+B)(x^2+x+1)+(Cx+D)=(x^2+1)…………. (1)

Equating the coefficients of x^3 in (1), we get A = 0

Equating the coefficients of x^2 in (1), we get A+B=1

$\Rightarrow$0+B=1B=1

Equating the coefficients of x in (1), we get

A+B+C=0$\Rightarrow$0+1+C=0$\Rightarrow$C=-1

Equating the constants in (1), we get

B+D=1$\Rightarrow$1 + D = 1

$\Rightarrow$ D = 1 - 1

$\therefore \frac{x^2+1}{(x^2+x+1)^2}=\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{(x^2+x+1)^2}=\frac{1}{x^2+x+1}-\frac{x}{(x^2+x+1)^2}$

2) Resolve $\frac{1}{(x-1)^2(x-2)}$ into partial fractions

Sol: Let $\frac{1}{(x-1)^2(x-2)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x-2)}\Rightarrow\frac{1}{(x-1)^2(x-2)}$

$=\frac{A(x-1)(x-2)+B(x-2)+C(x-1)^2}{(x-1)^2(x-2)}$

$\Rightarrow$ A(x-1)(x-2)+B(x-2)+C$(x-1)^2$=1…..(1)

Putting x=1 in (1), we get A(0)+B(1-2)+C(0)=1$\Rightarrow$-B=1$\Rightarrow$B=-1

Putting x=2 in (1), we get A(0)+B(0)+C$(2-1)^2$=1$\Rightarrow$C=1

Equating the coefficients of x^2 in (1), we get A+C=0

$\Rightarrow$A=-C$\Rightarrow$A=-1

$\therefore \frac{1}{(x-1)^2(x-2)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x-2)}$

$= \frac{-1}{(x-1)} - \frac{1}{(x-1)^2} + \frac{1}{x-2}$

3) Find the coefficient of x^4 in the power series expansion of $\frac{3x}{(x-2)(x+1)}$

Sol: Resolving the given fraction into partial fraction

$$\frac{3x}{(x-2)(x+1)} = \frac{2}{x-2} + \frac{1}{x+1}$$

$$= \frac{2}{x-2} + \frac{1}{x+1} = \frac{2}{-2\left[1-\frac{x}{2}\right]} + \frac{1}{1+x}$$

$$= -\left[1-\frac{x}{2}\right]^{-1} + (1+x)^{-1} = -\left[1+\frac{x}{2}+\left(\frac{x}{2}\right)^2+\left(\frac{x}{2}\right)^3+\left(\frac{x}{2}\right)^4+.....\right]+\left[1-x+x^2-x^3+x^4+.....\right]$$

$\therefore$ coefficients of $x^4 = \frac{-1}{2^4} + 1 = 1 - \frac{1}{16} = \frac{15}{16}$

4) Resolve $\frac{x^2+5x+7}{(x-3)^3}$ into partial fractions

Sol: Put x-3=y then x=y+3

$$\therefore \frac{x^2+5x+7}{(x-3)^3} = \frac{(y+3)^2+5(y+3)+7}{y^3}$$

$$= \frac{y^2+6y+9+5y+15+7}{y^3} = \frac{y^2+11y+31}{y^3}$$

$$= \frac{1}{y} + \frac{11}{y^2} + \frac{31}{y^3} = \frac{1}{x-3} + \frac{11}{(x-3)^2} + \frac{31}{(x-3)^3}$$

5) Resolve $\frac{3x+7}{x^2-3x+2}$ into partial fractions

Sol: The denominator $x^2-3x+2=x^2-2x-x+2=x(x-2)-(x-2)=(x-1)(x-2)$

G.E. = $\frac{3x+7}{x^2-3x+2} = \frac{3x+7}{(x-1)(x-2)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$

$\Rightarrow$ A(x-2)+B(x-1)=3x+7…….(1)

Putting x=1 in (1) we get A(1-2)+B(1-1)

=3(1)+7 $\Rightarrow$ -A=10 $\Rightarrow$ A=-10

Putting x=2 in (1), we get A(x-2)+B(2-1)=3(2)+7 $\Rightarrow$ B=13

$$\therefore \quad \frac{3x+7}{x^2-3x+2}=\frac{A}{(x-1)}+\frac{B}{(x-2)}=\frac{-10}{x-1}+\frac{13}{x-2}$$

6) Resolve $\frac{1}{(x-1)^2(x-2)}$ into partial fractions

Sol: Let $\frac{1}{(x-1)^2(x-2)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x-2)}$

$$\Rightarrow \frac{1}{(x-1)^2(x-2)}$$

$$=\frac{A(x-1)-(x-2)+B(x-2)+C(x-1)^2}{(x-1)^2(x-2)}$$

$\Rightarrow$A(x-1)(x-2)+B(x-2)+C$(x-1)^2$=1……….. (1)

Putting x=1 in (1), we get A(0)+B(1-2)+C(0)=1$\Rightarrow$-B=1$\Rightarrow$B=-1

Putting x=2 in (1), we get A(0)+B(0)+C$(2-1)^2$=1$\Rightarrow$C=1

Equating the coefficients of x^2 in (1), we get

A+C=0$\Rightarrow$A=-C$\Rightarrow$A=-1

$$\therefore \frac{1}{(x-1)^2(x-2)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x-2)}$$

$$=\frac{-1}{(x-1)}-\frac{1}{(x-1)^2}+\frac{1}{x-2}$$

7) Resolve $\frac{x^3}{(x-a)(x-b)(x-c)}$ into partial fractions

Sol: Let $\frac{x^3}{(x-a)(x-b)(x-c)}=1+\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$ Here deg(Nr)=deg(Dr)

$$=\frac{(x-a)(x-b)(x-c)+A(x-b)(x-c)+B(x-a)(x-c)+C(x-a)(x-b)}{(x-a)(x-b)(x-c)}$$

$\Rightarrow$(x-a)(x-b)(x-c)+A(x-b)(x-c)+B(x-a)(x-c)+C(x-a)(x-b) = x^3 …….(1)

Putting x=a in (1) we get 0+A(a-b)(a-c)+0+0 =a^3

$$A=\frac{a^3}{(a-b)(a-c)}$$

Similarly by putting x=b and x=c we get $B=\frac{b^3}{(b-a)(b-c)}, C=\frac{c^3}{(c-a)(c-b)}$

$$\therefore \frac{x^3}{(x-a)(x-b)(x-6)} = 1 + \frac{a^3}{(a-b)(a-c)(x-a)} + \frac{b^3}{(b-c)(b-a)(x-b)} + \frac{c^3}{(c-a)(c-b)(x-c)}$$

8) Resolve $\frac{x^3}{(2x-1)(x+2)(x-3)}$ into partial fractions

Sol: Let $\frac{x^3}{(2x-1)(x+2)(x-3)} = \frac{1}{2} + \frac{A}{2x-1} + \frac{B}{x+2} + \frac{C}{x-3}$

$$= \frac{(2x-1)(x+2)(x-3) + A(2)(x+2)(x-3) + B(2)(2x-1)(x-3) + C(2)(2x-1)(x+2)}{2(2x-1)(x+2)(x-3)}$$

$\therefore (2x-1)(x+2)(x-3) + 2A(x+2)(x-3) + 2B(2x-1)(x-3) + 2C(2x-1)(x+2) = 2x^3$ ....(1)

Putting $x=\frac{1}{2}$ in (1) we get $0+2A\left[\frac{5}{2}\right]\left[-\frac{5}{2}\right]+B(0)+C(0)$

$$= 2\left[\frac{1}{8}\right] \Rightarrow \frac{-25A}{2} = \frac{1}{4} \Rightarrow A = \frac{-1}{50}$$

Putting x=-2 in (1), we get 0+A(0)+2B(-5)(-5)+C(0)

$$=2(-8) \Rightarrow 50B = -16 \Rightarrow B = \frac{-8}{25}$$

Putting x=3 in (1), we get 0+A(0)+B(0)+2C 5(5) = 2(27)

$$= 25C=27 \Rightarrow C = \frac{27}{25}$$

$$\therefore \frac{x^3}{(2x-1)(x+2)(x-3)} = \frac{1}{2} + \frac{A}{(2x-1)} + \frac{B}{(x+2)} + \frac{C}{(x-3)} = \frac{1}{2}$$

$$\frac{1}{56(2x-1)} - \frac{8}{25(x+2)} + \frac{27}{25(x-3)}$$

9. Resolve $\frac{x^3}{(2x-1)(x-1)^2}$ into partial fractions

Sol: Let $\frac{x^3}{(2x-1)(x-1)^2} = \frac{1}{x} + \frac{A}{(2x-1)} + \frac{B}{x-1^3} + \frac{C}{(x-1)^2}$

$$\frac{x^3}{(2x-1)\,(x-1)^2} = \frac{(2x-1)(x-1)^2 + A(2)(x-1)^2 + B(2)(2x-1)(x-1) + C(2)(2x+1)}{2(2x-1)(x-1)^2}$$

Putting $x=\frac{1}{2}\bar{x}$ (1), we get

$2\left(\frac{1}{8}\right) = 2A\left(\frac{1}{4}\right)$

A = $\frac{1}{2}$

$\Rightarrow 2A+2B-2C=1 \Rightarrow 2B=1+2C-2A \Rightarrow 2B=1+2-1 \Rightarrow 2B=2 \Rightarrow B=1$

$\therefore \frac{x^3}{(2x-1)(x-1)^2} = \frac{1}{2} + \frac{A}{2x-1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{1}{2} + \frac{1}{2(2x-1)} + \frac{1}{(x-1)} + \frac{1}{(x-1)^2}$

10) Resolve $\frac{x+3}{(1-x)^2(1+x^2)}$ into partial fractions

Sol: Let $\frac{x+3}{(1-x)^2(1+x^2)} = \frac{A}{(1-x)} + \frac{B}{(1-x)^2} + \frac{Cx+D}{(1+x^2)}$

= $\frac{A(1-x)(1+x^2)+B(1+x^2)+(Cx+D)(1-x)^2}{(1-x)^2(1+x^2)}$

$\therefore A(1-x)(1+x^2)+B(1+x^2)+(cx+D)(1-x)^2 = x+3$

$\Rightarrow A(1-x)(1+x^2)+B(1+x^2)+(cx+D)(x^2-2x+1)=x+3.......(1)$

Putting x=1 in (1) we get A(0)+B(1+1)+(Cx+D)(0)

=1+3 $\Rightarrow$ 2B=4 $\Rightarrow$ B=2………(2)

Comparing the coefficients of x^3 in (1), we get –A+C=0 $\Rightarrow$ A=C………(3)

Comparing the constant terms in (1), we get A+B+D=3

$\Rightarrow$ A+0=3-B=3-2=1 $\Rightarrow$ A+D=1……..(4)

Comparing the coefficients of x^2 is (1) we get A+B-2C+D=0

$\Rightarrow$ 2C=(A+D)+B=1+2=3 $\Rightarrow$ C= $\frac{3}{2}$

From (3), A=C= $\frac{3}{2}$; from (4) D=1-A=1- $\frac{3}{2}$ = $\frac{-1}{2}$

$\therefore \frac{x+3}{(1-x)^2(1+x^2)} = \frac{A}{(1-x)} + \frac{B}{(1-x)^2} + \frac{Cx+D}{(1+x^2)} = \frac{3}{2(1-x)} + \frac{2}{(1-x)^2} + \frac{(3x-1)}{2(1+x^2)}$

Chapter-8

MEASURES OF DISPERSION

Weightage: (2 + 7)

Very short answer questions (2 M)

Level-1

Find the mean deviation about mean of the following discrete data

3, 6, 10, 4, 9, 10

Sol: Let $\bar{x}$ be the mean of given data

$$\bar{x} = \frac{\text{sum of observations}}{\text{number ofobservation}}$$

$$\bar{x} = \frac{3+6+10+4+9+10}{6} = \frac{42}{6} = 7$$

Now calculation of mean duration from mean

x_i	3	6	10	4	9	10
$\lvert x_i - \bar{x} \rvert$	4	1	3	3	2	3

Total $\sum \lvert x_i - \bar{x} \rvert$ = 4 + 7 + 3 + 3 + 2 + 3 = 16

$\therefore$ mean deviation from mean = $\dfrac{\sum_{i-1}^{n} \lvert x_i - \bar{x} \rvert}{n} = \dfrac{16}{6} = \dfrac{8}{3} =$ 2.67

2. Find the mean deviation about median for the following data 13, 17, 16, 11, 13, 10, 16, 11, 18, 12, 17

Sol: Arranging data in ascending order 10, 11, 11, 12, 13, 13, 16, 16, 17, 17, 18

Here number of observations n = 11 (odd)

(i.e.,) Median = $\dfrac{n+1}{2} = \dfrac{11+1}{2} = 6$

$\therefore$ Median (b) = 6^{th} observation is 13

Ungrouped data:

Median = $\dfrac{n+1}{2}$ if n is odd

Calculation of mean deviation about median

x_i	10	11	12	13	13	16	16	17	17	18	
$\|x_i - b\|$	3	22	1	0	0	3	3	4	4	5	$\sum\|x_i - b\| = 27$

∴ Mean deviation about median

$$\frac{\sum_{i=1}^{n}|x_i - b|}{n} = \frac{27}{11} = \boxed{2.45}$$

3. Find the variance and standard deviation of following data 5, 12, 8, 18, 6, 8, 2, 10

Sol. The mean of given data is $\bar{x} = \frac{5+12+3+18+6+8+2+10}{8} = \frac{64}{8} = 8$

$\bar{x_i}$	5	12	3	18	6	8	2	10
$x_i - \bar{x}$	-3	4	-5	10	-2	0	-6	2
$\left(x_i - \bar{x}\right)^2$	9	16	25	100	4	0	36	4

$\sum_{i=1}^{n}(x_i - \bar{x})^2 = 194$

∴ Variance $\boxed{\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}$

$$\sigma^2 = \frac{194}{8} = \boxed{24.25}$$

Standard deviation (σ) = $\sqrt{24.25}$ = $\boxed{4.92}$ (approx)

4. Find the mean deviation from mean of the following discrete data 6, 7, 10, 12, 13, 4, 12, 16.

Sol. The mean of given data is $\bar{x} = \frac{6+7+10+12+13+4+12+16}{8} = 10$

The absolute value of deviations $\left|x_i - \bar{x}\right|$ = 4, 3, 0, 2, 3, 6, 2, 6

$\boxed{3.25}$

Mean deviation from mean = $\frac{\sum_{i=1}^{n}\left|x_i - \bar{x}\right|}{n} = \frac{4+3+0+2+3+6+2+6}{8} = \frac{26}{8}$

Level-II

5. The coefficient of variation of two distribution are 60 and 70 and their standard deviations are 21 and 16 respectively. Find their arithmetic means.

Solu: Given coefficient of variation of 1st distribution = 60

Coefficient of variation of 2nd distribution = 70

And standard deviations $\sigma_1 = 21, \sigma_2 = 16$

Co-efficient of variation = $\frac{\sigma}{\overline{x}}$ x 100 → arithmetic mean

For 1st distribution $60 = \frac{21}{\overline{x}_1} \times 100 \Rightarrow \overline{x}_1 = \frac{21 \times 100}{60}$

$\overline{x}_1 = 35$

For 2nd distribution $70 = \frac{16}{\overline{x}_2} = 100 \Rightarrow \overline{x}_2 = \frac{16 \times 100}{70}$

$\overline{x}_2 = \frac{160}{7} = 22.85$

6. Find the mean deviation about mean for the following data

X_i	**2**	**5**	**7**	**8**	**10**	**35**
f_i	**6**	**8**	**10**	**6**	**8**	**2**

Sol. Calculation of mean deviation about mean.

x_i	f_i	$f_i x_i$	$\lvert x_i - \overline{x} \rvert = \lvert x_i - 8 \rvert$	$f_i \lvert x_i - \overline{x} \rvert$
2	6	12	6	36
5	8	40	3	24
7	10	70	1	10
8	6	48	0	0
10	8	80	2	16
35	2	70	27	54
	N=$\sum f_i = 40$	$\sum f_i x_i = 320$		N=$\sum f_i \lvert x_i - 8 \rvert = 140$

Mean $\boxed{\bar{x} = \frac{\sum f_i x_i}{\sum f_i}}$ $\quad \therefore \bar{x} = \frac{320}{40} = 8$

Mean deviation about mean = $\boxed{\frac{1}{N}\sum_{i=1}^{n} f_i \left| x_i - \bar{x} \right|}$

$= \frac{140}{40} = \boxed{3.5}$

7. Find the variance of the data 6, 7,10, 12, 13, 4, 8, 12

Soln: Mean $\bar{x} = \frac{6+7+10+12+13+4+8+12}{8} = \frac{72}{8} = 9$

X_i	6	7	10	12	13	4	8	12	
$x_i - \bar{x}$	- 3	-2	1	3	4	-5	-1	3	
$\left(x_i - \bar{x}\right)^2$	9	4	1	9	16	25	1	9	$\sum (x_i - \bar{x})^2 = 74$

$\therefore$ variance = $\frac{\sum (x_i - \bar{x})^2}{8} = \frac{74}{8} = \boxed{9.25}$

8. The variance of 20 observations is 5. If each is multiplied by 2, find the variance of resulting observations.

Solu: We know that each observation in data is multiplied by k then variance of resulting data = k^2 times original variance.

$\therefore$ variance of new observations = $2^2.5 = 20$

Long Answer Questions: (7 M)

Level – I

1. Find the mean deviation from mean of following data, using stem deviation method.

Marks:	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**
No. of students:	**6**	**5**	**8**	**15**	**7**	**6**	**3**

Solu: We shall construct the following table.

Class interval	Midpoint (x_i)	No. of students (f_i)	$y_i = \frac{x_i - A}{h}$ $y_i = \frac{x_i - 35}{10}$	$f_i y_i$	$\lvert x_i - \bar{x} \rvert$ $\lvert x_i - 33.4 \rvert$	$f_i \lvert x_i = \bar{x} \rvert$
6-10	6	6	-3	-18	28.4	170.4
10-20	15	5	-2	-10	18.4	92
20-30	25	8	-1	-8	8.4	67.2
30-40	35	15	0	0	1.6	24.0
40-50	45	7	1	7	11.6	81.2
50-60	55	6	2	12	21.6	129.6
60-70	65	3	3	9	31.6	94.8
		N=50		= - 8 $\sum f_i y_i = -8$		659.2

Here width of class interval is 10 and assumed mean A = 35

Here N = 50

Mean $\bar{x} = A + \left(\frac{\sum f_i y_i}{N} \right) h$

$\therefore \bar{x} = 35 + \left(\frac{-8}{50} \right) \times 10 =$ 33.4

Mean deviation from mean = $\frac{1}{N} \sum f_i \lvert x_i - \bar{x} \rvert$

$= \frac{1}{50}$ (659.2) = 13.18 approx)

3. Calculate the variance and standard deviation of following continuous frequency distribution.

Class Interval	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
Frequency	3	7	12	15	8	3	2

Sub: Constructing the table with given data.

Class Interval	**Frequency (f_i)**	**Midpt (x_i)**	$yi=\frac{xi-65}{10}$	yi^2	**fiyi**	**fiyi²**
30-40	3	35	-3	9	-9	27
40-50	7	45	-2	4	-14	28
50-60	12	55	-1	1	12	12
60-70	15	**65**	0	0	0	0
70-80	8	75	1	1	8	8
80-90	3	85	2	4	12	12
90-100	2	95	3	9	18	18

N = 50 $\sum fiyi = -15$ $\sum fiyi^2 = 105$

Here width of class interval (h) = 10, assumed mean A = 65

Mean = $\bar{x} = A + \left(\frac{\sum fiyi}{N}\right) x\ h$ = 65+ $\frac{(-15)}{50} x\, 10$ = 62 (1M)

Variance $\sigma_x^2 = \frac{h^2}{N^2}\left[N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right]$

$$\sigma_x^2 = \frac{100}{2500}\left[50(105) - (-15)^2\right]$$

$$= \frac{1}{25}[5250 - 225] = \boxed{201}$$

$\therefore$ Standard deviation approx = $\sigma_x = \sqrt{201} = \boxed{14.18}$

3. Calculate the variance and standard deviation for the discrete frequency distribution.

x_i	4	8	11	17	20	24	32
F_i	3	5	9	5	4	3	1

Sol: Construction of table

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	12	-10	100	300

8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
	N = 30	$\sum f_i x_i$ =420			$\sum f_i\left(x_i-\bar{x}\right)^2$ =1374

Here N = 30, $\sum_{i=1}^{7} f_i x_i = 420$ $\sum_{i=1}^{7} f_i\left(x_i-\bar{x}\right)^2 = 1374$

$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{420}{30} = \boxed{14}$

Variance $\sigma^2 = \frac{1}{N}\sum fi(x_i-\bar{x})^2 = \frac{1}{30}(1374) = \boxed{45.8}$

Standard deviation T= $\sqrt{45.8} = \boxed{6.77}$

4. Find the mean deviation about mean for the following continuous distribution.

Height (in cm)	**95-105**	**105-115**	**115-125**	**125-135**	**135-145**	**145-155**
No. of boys	**9**	**13**	**26**	**30**	**12**	**10**

Solution: Construction of table

Height	**No. of boys f_i**	**Midpoint X_i**	**$F_i x_i$**	**$\lvert x_i - \bar{x}\rvert$**	**$f_i\lvert x_i - \bar{x}\rvert$**
95-105	9	100	900	25.3	227.7
105-115	13	110	1430	15.3	198.9
115-125	26	120	3120	5.3	137.8
125-135	30	130	3900	4.7	141.0
135-145	12	140	1680	14.7	176.4
145-155	10	150	1500	24.7	247.0

N = $\sum f_i$ = 100 $\sum f_i x_i$ = 12530 $\sum f_i\left|x_i-\bar{x}\right| = (1128.8)$

$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{12530}{100} = \boxed{125.3}$

Mean deviation from mean = $\frac{1}{N}\sum f_i |x_i - \bar{x}|$

$= \frac{1}{100}(1128.8) = \boxed{11.29}$ (approx)

Level-2

5. The complete table gives the daily wages of workers in a faculty compute the standard deviation and coefficient of variation of wages of workers.

Wage (Rs)	**125-175**	**175-225**	**225-275**	**275-325**	**325-375**	**375-425**
No. of wages	2	22	19	14	3	4

Wages (Rs)	**425-475**	**475-525**	**525-575**
No. of workers	6	1	1

Solution:

Class interval	**Midpt x_i**	**Frequency f_i**	**$y_i = \frac{x_i - A}{h}$**	**$f_i y_i$**	y_i^2	$f_i y_i^2$
125-175	150	2	-3	-6	9	18
175-225	200	22	-2	-44	4	88
225-275	250	19	-1	-19	1	19
275-325	300	14	0	0	0	0
325-375	350	3	1	3	1	3
375-425	400	4	2	8	4	16
425-475	450	6	3	18	9	54
475-525	500	1	4	4	16	16
525-575	550	1	5	5	25	25
		N=7		$\sum f_i y_i = -31$		$\sum f_i y_i^2 = 239$

Here width of class interval (h) = 50, assumed mean A = 300

$$\boxed{\bar{x} = A + \left(\frac{\sum f_i y_i}{N}\right) X\ h}$$

Mean $= 300 + \left(\frac{-31}{72}\right)50 = 300 - \frac{1550}{72} = \boxed{278.47}$

Variance $\boxed{\sigma_x^2 = \frac{h^2}{N^2}\left[N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right]}$

$\sigma_x = \sqrt{2500\left(\frac{239}{72} - \frac{961}{72 \times 72}\right)} = \boxed{88.52}$

Coefficient of variation $= \frac{\sigma_x}{\bar{x}} \times 100 = \frac{88.52}{278.47} \times 100$

$= \boxed{31.79}$

6. The mean of 5 observations is 4.4 their variance is 8.24. If three of the observations are 1, 2 and 6. Find the other two observations.

Solu. Let the other two observations be x and y. Then the series is 1, 2, 6, x, y.

$\boxed{\bar{x} = \frac{\text{Sum of observations}}{\text{No. of observations}}}$ $\therefore\ x^2 = \frac{9+x+y}{5}$

But mean $\bar{x}$ = 4.4 given

$\therefore\ \frac{9+x+y}{5} = 4.4 \Rightarrow x + y = 13 \longrightarrow (1)$

Variance $\sigma^2 = 8.24$

$\Rightarrow\ \frac{1}{n}\sum x_i^2 - \left(\bar{x}\right)^2 = 8.24$

$\Rightarrow\ \frac{1+4+36+x^2+y^2}{5} - (4.4)^2 = 8.24$

$\Rightarrow x^2 + y^2 = 5(8.24) + 5(19.36) - 41$

$\Rightarrow x^2 + y^2 = 97 \longrightarrow (2)$

$(2) \Rightarrow (x + y)^2 - 2xy = 97$

$(13)^2 - 2xy = 97 \Rightarrow 2xy = 72$

xy = 36

Then x + y = 13, xy = 36

$(x-y)^2 = (x+y)^2 - 4xy = 169 - 144 = 25$

x – y = 5 solving x + y = 13 and x – y = 5 We get $\boxed{X = 9, y = 4}$

Chapter-9

PROBABILITY

Weightage: (4 + 4 + 7)

***Short Answer Type Questions*(4 Marks) :**

LEVEL-1:

1) **A speaks truth in 75% of cases and B in 80% cases. What is the probability that their statements about incident do not match.**

Sol: Let E_1 and E_2 be the events that A and B speak truth respectively

$\therefore P(E_1)=\frac{75}{100}=\frac{3}{4} \Rightarrow P(\overline{E_1})=1-\frac{3}{4}=\frac{1}{4}$

$P(E_2)=\frac{80}{100}=\frac{4}{5} \Rightarrow P(\overline{E_2})=1-\frac{4}{5}=\frac{1}{5}$ $\boxed{P(A)+P(\overline{A})=1}$ (1M)

$\therefore$ The probability that their statements about an incident do not match= $P(E_1 \cap \overline{E_2})+P(\overline{E_1} \cap E_2)$

$= P(E_1).P(\overline{E_2})+p(\overline{E_1}).P(E_2)$ ($\because$ E_1,E_2 are independent events) **(1M)**

$= \frac{3}{4}.\frac{1}{5}+\frac{1}{4}.\frac{4}{5}=\boxed{\frac{7}{20}}$ **(2M)**

2) **A,B,C are three horses in a race. The probability of A to win the race is twice that of B and probability of B is twice that of C. What are the probabilities of A, B and C to win the race.**

Sol: Let A, B, C be three horses representing as events A, B, C respectively

Given P(A) = 2P(B) $\rightarrow$ (1) **(1M)**

P(B) = 2P(C) $\rightarrow$ (2)

Let P(C) = x

$\therefore$ P(B) = 2x and P(A) = 2(2P(C))

= 4P(C) **(1M)**

= 4x

We know P(A) + P(B) + P(C) = 1

$\therefore 4x+2x+x=1 \Rightarrow 7x=1$ or $x=\frac{1}{7}$

(i. e. $\boxed{P(C)=\frac{1}{7}}$, $\boxed{P(B)=2x=\frac{2}{7}}$ $\boxed{P(A)=4x=\frac{4}{7}}$ **(1M)**

3) A and B are events with P(A)=0.5, P(B)=0.4 and $P(A\cap B)=0.3$. Find the probability that (i) A does not occur (ii) neither A nor B occurs

Sol: (i) We know that A^C denotes the event : A does not occur and $(A\cup B)^C$ denotes the event : neither A nor B occurs. **(1M)**

Then $P(A^C)=1\text{-}P(A)=1\text{-}0.5=\boxed{0.5}$ **(1M)**

(ii) By addition theorem $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

= 0.5+0.4-0.3

=0.6 **(1M)**

$\therefore P(A\cup B)^C=1\text{-}P(A\cup B)$

= 1-0.6

= $\boxed{0.4}$ **(1M)**

4) A problem in calculus is given to two students A and B whose chances of solving it are $\frac{1}{3}$ and $\frac{1}{4}$. Find the probability of the problem being solved if both of them try independently.

Sol: Let E_1, E_2 be events of A and B to solve a problem in calculus independently.

$\therefore P(E_1)=\frac{1}{3}, P(\overline{E_1})=\frac{2}{3}$ $\boxed{\because P(A)+P(\overline{A})=1}$

$P(E_2)=\frac{1}{4}, P(\overline{E_2})=\frac{3}{4}$ **(1M)**

$\therefore$ The probability of problem being solved

= 1-probability that the problem will not be solved

= $1\text{-}P(\overline{E_1}\cap\overline{E_2})$ **(1M)**

$=1\text{-}P(\overline{E_1}).P(\overline{E_2})$

$=1-\frac{2}{3}.\frac{3}{4}=\boxed{\frac{1}{2}}$ **(2M)**

5) Find the probability of drawing on Ace or a spade from a well shuffled pack of 52 playing cards.

Sol: The number of ways of selecting a card from a pack of 52 cards is ${}^{52}C_1=52$ **(1M)**

Let A, B be events of drawing an ace and spade respectively

$P(A)=\frac{4}{52}$ ($\because$ 4 aces in a pack)

$P(B)=\frac{13}{52}$ ($\because$ 13 spades in a pack)

$P(A\cap B)=\frac{1}{52}$ ($\because$ only one ace in 13 spade cards) **(1M)**

$\therefore$ Probability of drawing an ace or spade is $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore P(A\cup B)=\frac{4}{52}+\frac{13}{52}-\frac{1}{52}=\frac{16}{52}=\boxed{\frac{4}{13}}$ **(2M)**

6) The probability for a contractor to get a road contract is $\frac{2}{3}$ and to get a building contract is $\frac{5}{9}$. The probability to get atleast one contract is $\frac{4}{5}$. Find the probability that he gets both the contracts.

Sol: Let A be event of getting road contract B be event of getting building contract.

Given $P(A)=\frac{2}{3},P(B)=\frac{5}{9}$ **(1M)**

P(atleast one) = $P(A\cup B)=\frac{4}{5}$ **(1M)**

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore \frac{4}{5}=\frac{2}{3}+\frac{5}{9}-P(A\cap B)$

$\therefore P(A\cap B)=\frac{11}{9}-\frac{4}{5}=\frac{19}{45}$ **(2M)**

Level:2

7) **Let A and B be independent events with P(A)=0.2, P(B)=0.5. Find (i)** $P\left(\frac{A}{B}\right)$ **(ii)** $P\left(\frac{B}{A}\right)$ **(iii)** $P(A\cap B)$ **(iv)** $P(A\cup B)$

Sol: (i) Given A and B are independent events $P(A\cap B)=P(A)P(B)$

(i) $\boxed{P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}}$

$P\left(\frac{A}{B}\right)=\frac{P(A).P(B)}{P(B)}=P(A)=0.2$ **(1M)**

(ii) $\boxed{P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}}$

$P\left(\frac{B}{A}\right)=\frac{P(A).P(B)}{P(A)}=P(B)=0.5$ **(1M)**

(iii) $P(A\cap B)=P(A).P(B)$

=(0.2)(0.5)

= 0 .1 **(1M)**

(iv) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

= 0.2+0.5-0.1 = 0.6

(1M)

8) The probability that Australia wins a match against India in a cricket game is given to be $\frac{1}{3}$. If India and Australia play 3 matches what is the probability that

(i) Australia will lose all three matches

(ii) Australia will win atleast one match

Sol: Let A be an event that Australia wins a match against India in a cricket game

$\therefore P(A)=\frac{1}{3} \Rightarrow P(\overline{A})=\frac{2}{3}$ $\because P(\overline{A})=1-P(A)$ **(1M)**

(i) Probability that Australia will loose all three matches = $P(\overline{A}).P(\overline{A}).P(\overline{A})$

$= \frac{2}{3}.\frac{2}{3}.\frac{2}{3} = \frac{8}{27}$ **(1M)**

(ii) Probability that Australia will win atleast one match = 1-probability of loosing all matches

$= 1-\frac{8}{27}=\frac{19}{27}$ **(1M)**

9) A bag contains 12 two rupee coins, 7 one rupee coins, 4 half a rupee coins. If three coins are selected at random then find the probability that

(i) Sum of three coins is maximum (ii) Sum of three coins is minimum

(iii) each coin is of different value

Sol: The sample space of the experiment getting 3 coins from 23 coins $n(S)={}^{23}C_3$

(1M)

(i) Even A: getting sum maximum

Select 3 coins from 12 (2Rs coins) in ${}^{12}C_3$ ways

$\therefore n(A)={}^{12}C_3$

$P(A)=\frac{{}^{12}C_3}{{}^{23}C_3}$ **(1M)**

(ii) Event B : getting sum minimum

Select 3 coins from 4($\frac{1}{2}$ Rs coins) in ${}^{4}C_{3}$ ways

$\therefore n(B)={}^{4}C_{3}$

$\boxed{P(B)=\frac{{}^{4}C_{3}}{{}^{23}C_{3}}}$ **(1M)**

(iii) Event C: each one is of different values

Select 1 coin from 12 (2 Rs coins) in ${}^{12}C_{1}$ ways

Select 1 coin from 7 (1 Rs coins) in ${}^{7}C_{1}$ ways

Select 1 coin from 4 ($\frac{1}{2}$ Rs coins) in ${}^{4}C_{1}$ ways

$\therefore n(C)={}^{12}C_{1}.{}^{7}C_{1}.{}^{4}C_{1}$

$\therefore \boxed{P(C)=\frac{{}^{12}C_{1}.{}^{7}C_{1}.{}^{4}C_{1}}{{}^{12}C_{3}}}$ **(1M)**

10) The probability of three events A,B,C are such that P(A)=0.3, P(B)=0.4, P(C)=0.8, $P(A\cap B)$ **=0.08,** $P(A\cap C)$ **=0.28,** $P(A\cap B\cap C)$ =0.09 and $P(A\cup B\cup C)\geq 0.75$. Show that $P(B\cap C)$ lies in interval [0.23, 0.48]

Sol: Given $P(A)=0.3, P(B)=0.4, P(C)=0.8$

$P(A\cap B)=0.08, P(A\cap C)=0.28, P(A\cap B\cap C)=0.09$

$P(A\cup B\cup C)\geq 0.75$

$\boxed{\because 0\leq P(A)\leq 1}$

Clearly $0.75\leq P(A\cup B\cup C)\leq 1$

$\therefore P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$

$\Rightarrow 0.75\leq 0.3+0.4+0.8-0.08-P(B\cap C)-0.28+0.09\leq 1$

$\Rightarrow 0.75\leq 1.23-P(B\cap C)\leq 1$

$\Rightarrow 0.75-1.23 \leq -P(B \cap C) \leq 1-1.23 \Rightarrow 0.48 \geq P(B \cap C) \geq 0.23$

$\Rightarrow 0.23 \leq P(B \cap C) \leq 0.48$

$\therefore P(B \cap C) \in [0.23, 0.48]$

11) **Bag B_1 contains 4 white and 2 black balls. Bag B_2 contains 3 white and 4 black balls. A bag is drawn at random and a ball is chosen at random. What is the probability that the ball drawn is white**

Sol: Let E_1, E_2 be events of choosing bags B_1 and B_2 resp. Then $P(E_1)=P(E_2)=\frac{1}{2}$

$P\left(\frac{W}{E_1}\right)=\frac{4}{6}=\frac{2}{3}$ $\quad P\left(\frac{W}{E_2}\right)=\frac{3}{7}$ **(1M)**

$W=(W \cap E_1) \cup (W \cap E_2)$ and $(W \cap E_1) \cap (W \cap E_2)=\phi$ **(1M)**

$\therefore P(W)=P(W \cap E_1)+P(W \cap E_2)$

$P(E_1).P\left(\frac{W}{E_1}\right)+P(E_2).P\left(\frac{W}{E_2}\right)$ [by multiplication theorem]

$=\frac{1}{2}.\frac{2}{3}+\frac{1}{2}.\frac{3}{7}=\frac{1}{3}+\frac{3}{14}=\boxed{\frac{17}{42}}$ **(2M)**

Long Answer Questions **(7 Marks)**

Level:1

1) **If A,B,C are three independent events of an experiment such that** $P(A \cap B^C \cap C^C)=\frac{1}{4}, P(A^C \cap B \cap C^C)=\frac{1}{8}, P(A^C \cap B^C \cap C^C)=\frac{1}{4}$ **, then find P(A), P(B) and P(C).**

Sol: Given $P(A \cap B^C \cap C^C)=\frac{1}{4} \rightarrow$ (1)

$P(A^C \cap B \cap C^C)=\frac{1}{8} \rightarrow$ (2)

$P\left(A^C \cap B^C \cap C^C\right)=\frac{1}{4} \rightarrow$ (3)

If A,B,C are independent events then $\left(P\cap B\cap C\right)=P(A).P(B).P(C)$ **(1M)**

$$\frac{(2)}{(3)}=\frac{P\left(A^C\right).P(B).P\left(C^C\right)}{P\left(A^C\right).P\left(B^C\right).P\left(C^C\right)}=\frac{\frac{1}{8}}{\frac{1}{4}}=\frac{1}{2}$$

$$\Rightarrow \frac{P(B)}{P\left(B^C\right)}=\frac{1}{2} \Rightarrow 2P(B)=P\left(B^C\right)$$

$$\Rightarrow 2P(B)=1-P(B)$$

$$\Rightarrow 3P(B)=1$$

$$\therefore P(B)=\frac{1}{3}$$ **(2M)**

$$\frac{(1)}{(2)}=\frac{P(A).P\left(B^C\right).P\left(C^C\right)}{P\left(A^C\right).P\left(B^C\right).P\left(C^C\right)}=\frac{\frac{1}{4}}{\frac{1}{4}}=1$$

$$\Rightarrow \frac{P(A)}{P\left(A^C\right)}=1 \Rightarrow P(A)=P\left(A^C\right)$$

$$\Rightarrow P(A)=1-P(A)$$

$$2P(A)=1$$

$$\therefore P(A)=\frac{1}{2}$$ **(2M)**

From (3) $P\left(A^C\right).P\left(B^C\right).P\left(C^C\right)=\frac{1}{4}$

$$\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right).P\left(C^C\right)=\frac{1}{4}$$

$$\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)P\left(C^C\right)=\frac{1}{4} \Rightarrow P\left(C^C\right)=\frac{3}{4}$$

$$\therefore P(C)=1-P\left(C^C\right)$$

$$=1-\frac{3}{4}$$

$$\Rightarrow \boxed{P(C)=\frac{1}{4}}$$ **(2M)**

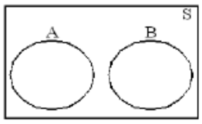
2) State and prove addition theorem on probability

Sol: **Statement:** Let A, B be any two events of a random experiment and P is a probability functions

then $P(A \cup B)=P(A)+P(B)-P(A \cap B)$

Prof: Let A and B are any two events of random experiment and P is a probability function

Case(i) : Suppose $A \cap B=\phi$ then $P(A \cap B)=P(\phi)=0$

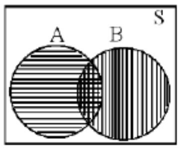


Now $P(A \cup B)=P(A)+P(B)$

$\Rightarrow P(A \cup B)=P(A)+P(B)-0$

$\therefore P(A \cup B)=P(A)+P(B)-P(A \cap B)$ **(1M)**

Case(ii): Suppose $A \cap B=\phi$ then $A \cup B=A \cup (B-A)$ and $A \cup B=A \cup (B-A)$ and $A \cap (B-A)=\phi$



Now $P(A \cup B)=P[A \cup (B-A)]$

$=P(A)+P(B-A)$ **(1M)**

$(\because A \cap (B-A)=\phi)$

$=P(A)+P\left[B-(A \cap B)\right]$ $(\because B-A=B-(A \cap B))$ **(1M)**

$=P(A)+P(B)-P(A \cap B)$

$(\because \text{If } E_1 \subseteq E_2 \text{ then } P(E_2-E_1)=P(E_2)-P(E_1))$

$\therefore$ From Case (i) and Case (ii)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(2M)

3) A,B,C are 3 newspapers from a city 20% of the population read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C and 2% read all three. Find the percentage of population who read atleast one newspaper.

Sol: Let A,B,C are events of reading 3 newspaper A,B,C respectively

$$P(A) = \frac{20}{100} \quad P(B) = \frac{16}{100} \quad P(C) = \frac{14}{100}$$

$$P(A \cap B) = \frac{8}{100}, P(A \cap C) = \frac{5}{100}, P(B \cap C) = \frac{4}{100}$$ **(2M)**

$$P(A \cap B \cap C) = \frac{2}{100}$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$ **(1M)**

$$= \frac{20}{100} + \frac{16}{100} + \frac{14}{100} - \frac{8}{100} - \frac{5}{100} - \frac{4}{100} + \frac{2}{100}$$ **(4M)**

$$= \frac{52}{100} - \frac{17}{100} = \frac{35}{100}$$

$\therefore$ Percentage of population that read atleast one newspaper is 35%

4) State and prove Baye's theorem

Sol: **Statement :** Let E_1, E_2,……En be n mutually exclusive and exhaustive events of a random experiment with $P(E_i) \neq 0$ where i = 1,2,……n then for any event A of random experiment with $P(A) \neq 0$

$$P\left(\frac{E_k}{A}\right) = \frac{P(E_k).P\left(\frac{A}{E_k}\right)}{\sum_{i=1}^{n} P(E_i).P\left(\frac{A}{E_i}\right)} \text{ for } k=1,2,.....n$$ **(2M)**

Proof:Let S be sample space of random experiment. Let E_1, E_2,…………E_n be n mutually exclusive and exhaustive events of a random experiment with $P(E_i) \neq 0$

$\therefore E_i \cap E_j = \phi$ for $i \neq j$ [$\because$ events are mutually exclusive]

(1M)

Also $S=\bigcup_{i=1}^{n} E_i$ ($\because$ events are exhaustive events)

Let A be any event of the experiment, then $A=A\cap S$

$$\Rightarrow A=A\bigcap\left(\bigcup_{i=1}^{n} E_i\right) \Rightarrow A=\bigcup_{i=1}^{n}(A\bigcap E_i) \qquad [\because A\cap(B\cup C)=(A\cap B)\cup(A\cap C)]$$

Now $P(A)=P\left[\bigcup_{i=1}^{n}(A\cap E_i)\right]$

$$\Rightarrow P(A)=\sum_{i=1}^{n}P(A\cap E_i) \qquad [\because (A\cap E_i)\cap(A\cap Ej)=\phi \text{ for } i\neq j]$$

$$\Rightarrow P(A)=\sum_{i=1}^{n}P(E_i).P\left(\frac{A}{E_i}\right) \quad \rightarrow (1)$$ (by multiplication theorem)

(2M)

Now for any event E_k, $P\left(\frac{E_k}{A}\right)=\frac{P(E_k\cap A)}{P(A)}$ (by conditional prob)

$$\Rightarrow P\left(\frac{E_k}{A}\right)=\frac{P(E_k).P\left(\frac{A}{E_k}\right)}{\sum_{i=1}^{n} P(E_i).P\left(\frac{A}{E_i}\right)}$$ ($\because$ from (1)) **(2M)**

5) Three boxes B_1,B_2.B_3 contain balls with different colours as shown below

Box	**White**	**Black**	**Red**
B_1	**2**	**1**	**2**
B_2	**3**	**2**	**4**
B_3	**4**	**3**	**2**

A die is thrown B_1 is chosen if either 1 or 2 turns up B_2 is chosen if 3 or 4 turns up, B_3 is chosen if 5 or 6 turns up. Having chosen a box in this way, a ball is chosen at random from this box. If the ball drawn is found to be red, find the probability that it is from box B_2.

Sol: Let A_1, A_2, A_3 be events of selecting boxes B_1, B_2, B_3 resp.

Since B_1 is chosen either 1 or 2 turns up on die

$$P(A_1)=\frac{2}{6}=\frac{1}{3}$$

Since B_2 is chosen either 3 or 4 turns up on die

$$P(A_2)=\frac{2}{6}=\frac{1}{3}$$

Since B_3 is either 5 or 6 turns up

(1M)

$$\therefore P(A_3)=\frac{2}{6}=\frac{1}{3}$$

Let R be event of drawing Red ball from box B_2

(1M)

$$P\left(\frac{R}{A_1}\right)=\frac{2}{5},\ P\left(\frac{R}{A_2}\right)=\frac{4}{9},\ P\left(\frac{R}{A_3}\right)=\frac{2}{9}$$

Prob.of Red ball from box B_2 $P\left(\frac{A_2}{R}\right)=\dfrac{P(A_2).P\left(\frac{R}{A_2}\right)}{P(A_1).P\left(\frac{R}{A_1}\right)+P(A_2).P\left(\frac{R}{A_2}\right)+P(A_3).P\left(\frac{R}{A_3}\right)}$

(1M)

$$=\frac{\frac{1}{3}.\frac{4}{9}}{\frac{1}{3}.\frac{2}{5}+\frac{1}{3}.\frac{4}{9}+\frac{1}{3}.\frac{2}{9}}=\frac{\frac{1}{3}.\frac{4}{9}}{\frac{1}{3}\left[\frac{2}{5}+\frac{4}{9}+\frac{2}{9}\right]}$$

(4M)

$$P\left(\frac{A_2}{R}\right)=\frac{\frac{4}{9}}{\frac{48}{45}}=\frac{20}{48}=\boxed{\frac{5}{12}}$$

6) Three boxes numbered I, II, III contain the balls as follows

Box	White	Black	Red
I	1	2	3

II	2	1	1
III	4	5	3

One box is randomly selected and a ball is drawn from it. If the ball is red then find the probability that is is from box II

Sol: Let B_1,B_2,B_3 be events of selecting boxes I,II,III respectively

$$\therefore P(B_1)=\frac{1}{3}, P(B_2)=\frac{1}{3}, P(B_3)=\frac{1}{3}$$ **(1M)**

Let R be event of drawing red ball from box

$$P\left(\frac{R}{B_1}\right)=\frac{3}{6}=\frac{1}{2}, P\left(\frac{R}{B_2}\right)=\frac{1}{4}, P\left(\frac{R}{B_3}\right)=\frac{3}{12}=\frac{1}{4}$$ **(1M)**

The probability that red ball from box II is

$$P\left(\frac{B_2}{R}\right)=\frac{P(B_2).P\left(\frac{R}{B_2}\right)}{P(B_1).P\left(\frac{R}{B_1}\right)+P(B_2)P\left(\frac{R}{B_2}\right)+P(B_3).P\left(\frac{R}{B_3}\right)}$$ **(1M)**

$$\frac{\frac{1}{3}.\frac{1}{4}}{\frac{1}{3}.\frac{1}{2}+\frac{1}{3}.\frac{1}{4}+\frac{1}{3}.\frac{1}{4}}=\frac{\frac{1}{12}}{\frac{1}{6}+\frac{1}{12}+\frac{1}{12}}$$ **(1M)**

$$=\frac{\frac{1}{12}}{\frac{4}{12}}=\boxed{\frac{1}{4}}$$

Level:2

7) Define conditional probability. State and prove multiplication theorem on probability

Sol: **Conditional Probability :** Let A and B be two events of sample space with P(A) $\neq 0$ then the probability of B after event A has occurred is called conditional probability of B given A denoted by $P\left(\frac{B}{A}\right)$

(2M)

$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)},P(A)\neq 0$$

Multiplication theorem on Probability

Statement: Let A and B be two events of random experiment with P(A)>0 and P(B)>0 then

$$P(A\cap B)=P(A).P\left(\frac{B}{A}\right)=P(B).P\left(\frac{A}{B}\right)$$ **(2M)**

Proof: Let A and B be two events of random experiment with P(A)>0 and P(b)>0

From definition of contain probability

$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}\Rightarrow P(A\cap B)=P(A).P\left(\frac{B}{A}\right)\rightarrow(1)$$ **(1M)**

Also $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}\Rightarrow P(A\cap B)=P(B).P\left(\frac{A}{B}\right)\rightarrow(2)$

(1M)

From (1) and (2)

$$P(A\cap B)=P(A).P\left(\frac{B}{A}\right)=P(B).P\left(\frac{A}{B}\right)$$

(1M)

8) **In a shooting test the probability of A,B,C hitting the targets are $\frac{1}{2},\frac{2}{3}$ and $\frac{3}{4}$ respectively. If all of them fire at same target. Find the probability that (i) only one of them hits the target (ii) atleast one of them hits the target.**

Sol: Let A,B,C be events if hitting targets with A,B,C persons respectively.

$P(A)=\frac{1}{2}, P(B)=\frac{2}{3}, P(C)=\frac{3}{4}$

$$\boxed{P(\overline{A})=1\text{-}P(A)}$$

$P(\overline{A})=\frac{1}{2}, P(\overline{B})=\frac{1}{3}, P(\overline{C})=\frac{1}{4}$ **(1M)**

Clearly A,B,C are independent events

(i) Probability of only one of them hits target

$$= P\left(A\cap\overline{B}\cap\overline{C}\right)+P\left(\overline{A}\cap B\cap\overline{C}\right)+P\left(\overline{A}\cap\overline{B}\cap C\right)$$

$$= P(A).P\left(\overline{B}\right).P\left(\overline{C}\right)+P\left(\overline{A}\right).P(B).P\left(\overline{C}\right)+P\left(\overline{A}\right).P\left(\overline{B}\right).P(C)$$

$$=\frac{1}{2}.\frac{1}{3}.\frac{1}{4}+\frac{1}{2}.\frac{2}{3}.\frac{1}{4}+\frac{1}{2}.\frac{1}{3}.\frac{3}{4}$$

$$=\frac{1}{24}+\frac{2}{24}+\frac{3}{24}=\frac{6}{24}=\frac{1}{4}$$ **(3M)**

(ii) Probability of atleast one of them hits the target

= 1-probability of none of them hits the target

$$= 1-P\left(\overline{A}\cap\overline{B}\cap C\right)$$

$$=1-P\left(\overline{A}\right).P\left(\overline{B}\right).P\left(\overline{C}\right) \quad =1-\frac{1}{2}.\frac{1}{3}.\frac{1}{4}= 1-\frac{1}{24}=\frac{23}{24}$$

(3M)

9) Two persons A and B are rolling a die on the condition that the person who gets 3 first will win the game. If A starts the game, then find probabilities of A and B respectively to win the game.

Sol: Let P be probability of success and Q be probability of facture when rolling a die, getting 3 and not getting 3 respectively.

$P=\frac{1}{6}$ $Q=1-P=\frac{5}{6}$ **(1M)**

A B

P QP

QQP QQQP

P(A to win game) = P+QQP+QQQQP+…..

(1M)

P(A to win game) = $P+Q^2P+Q^4P+$…….

$=P[1+Q^2+Q^4.........\infty]$ **(3M)**

$= P\left(\frac{1}{1-Q^2}\right)$

$\because S_\infty = \frac{a}{1-r}$ GP

$= \frac{\frac{1}{6}}{1-\left(\frac{5}{6}\right)^2} = \boxed{\frac{6}{11}}$

(1M)

P(B to win game) = 1-P(A to win game)

$= 1-\frac{6}{11}$

$= \boxed{\frac{5}{11}}$ **(1M)**

10) Three urns have the following composition of balls

Urn-I : 1 White 2 Black

Urn-II : 2 White 1 Black

Urn-III: 2 White 2 Black

One of the urn is selected at random and a ball is drawn. It turns out to be white. Find the probability that it came from urn III

Sol: Let A_1, A_2, A_3 be events of selecting urns I,II,III

$P(A_1)=\frac{1}{3}\ P(A_2)=\frac{1}{3}\ P(A_3)=\frac{1}{3}$ **(1M)**

Let W be event of drawing white ball from an urn

$\therefore P\left(\frac{W}{A_1}\right)=\frac{1}{3}\ P\left(\frac{W}{A_2}\right)=\frac{2}{3}\ P\left(\frac{W}{A_3}\right)=\frac{2}{4}$ **(1M)**

Probability of white ball from urn III is

$$P\left(\frac{A_3}{W}\right)=\frac{P(A_3)P\left(\frac{W}{A_3}\right)}{P(A_1).P\left(\frac{W}{A_1}\right)+P(A_2)P\left(\frac{W}{A_2}\right)+P(A_3)P\left(\frac{W}{A_3}\right)}$$ **(1M))**

$$= \frac{\frac{1}{3}.\frac{2}{4}}{\frac{1}{3}.\frac{1}{3}+\frac{1}{3}.\frac{2}{3}+\frac{1}{3}.\frac{2}{4}} = \frac{\frac{1}{6}}{\frac{1}{9}+\frac{2}{9}+\frac{2}{12}} \quad = \quad 1/3$$

Chapter-10

RANDOM VARIABLES AND PROBABILITY DISTRIBUTIONS

Weightage (2 + 7)

***Very Short Answer Type Questions*(2 Marks) :**

1) If the mean and variance of a binomial variable X are 2.4 and 1.44 respectively, find $P(1<X\leq 4)$

Sol: Mean=2.4 $\Rightarrow$ np=2.4 $\rightarrow$ (1)

Variance=1.44 $\Rightarrow$ npq=1.44 $\rightarrow$ (2)

Mean=np Variance=npq

$\frac{(2)}{(1)} \Rightarrow q = 0.6$

$\because p = 1 - q = 0.4$ **(1M)**

From (1) n(0.4)=2.4 $\Rightarrow$ n=6

$P(X=r) = {}^{n}c_r p^r q^{n-r}$

$\therefore P(1<X\leq 4) = P(X=2)+P(X=3)+P(X=4)$

$= \sum_{r=2}^{4} {}^{6}c_r . (0.4)^r (0.6)^{6-r}$ **(1M)**

2) The probability that a person chosen at random is left handed (in handwriting) is 0.1. What is the probability that in a group of 10 people, there is one who is left handed

Sol: Here n=10, $p=\frac{1}{10}=0.1$ $\therefore q=1-p=0.9$ **(1M)**

To find P(x=1)

$p(x=k) = {}^{n}c_k p^k q^{n-k}$

$P(x=1) = {}^{10}c_1 p^1 q^9$ (Here k = 1)

$=10(0.1)(0.9)^9 = (0.9)^9$ **(1M)**

Sol: Given mean = np = 4 $\longrightarrow$ (1)

3) The mean and variance of a binomial distribution are 4 and 3 respectively. Find $P(x\geq 1)$

Variance =npq=3 $\longrightarrow$ (2)

Mean = np Variance = npq

$\frac{(2)}{(1)} \Rightarrow q=\frac{3}{4} \Rightarrow p=1-q$ **(1M)**

$=1-\frac{3}{4}=\frac{1}{4}$

From (1) $n\left(\frac{1}{4}\right)=4 \quad \therefore n=16$

$\boxed{P(x=k)={}^{n}c_k p^k q^{n-k}}$

$P(x \geq 1)$=1-P(x=0)

$=1-{}^{16}C_0\left(\frac{1}{4}\right)^0 \cdot \left(\frac{3}{4}\right)^{16} = \boxed{1-\left(\frac{3}{4}\right)^{16}}$

4) A Poisson variable satisfies P(x=1)=P(x=2), find P(x=5)

Sol: Given P(x=1)=P(x=2)

$\boxed{P(x=k)=\frac{e^{-\lambda}.\lambda^k}{\angle k}}$

$\Rightarrow \frac{e^{-\lambda}.\lambda^1}{\angle 1}=\frac{e^{-\lambda}.\lambda^2}{\angle 2}$

$\Rightarrow 1=\frac{\lambda}{2} \Rightarrow \boxed{\lambda=2}$ **(1M)**

$P(x=5)=\frac{e^{-\lambda}.\lambda^5}{\angle 5}=\frac{e^{-2}.(2)^5}{\angle 5}=\boxed{\frac{4}{15}e^{-2}}$ **(1M)**

Level-2

5) On an average rain falls on 12 days in every 30 days. Find the probability that, rain will fall on just 3 days of a given week.

Sol: Given $p=\frac{12}{30}=\frac{2}{5}$

$q=1-\frac{2}{5}=\frac{3}{5}$ Here n=7, r=3 **(1M)**

$\boxed{P(X=x)={}^{n}c_x.p^x q^{n-x}}$

Probability for rainfall on just 3 days is P(X=3)

$P(x=3)={}^{n}c_3.p^3 q^{n-3}$

$={}^{7}c_3\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^4$

$=\boxed{\frac{35\times 2^3\times 3^4}{5^7}}$ **(1M)**

6) For a binomial distribution with mean 6 and variance 2, find the first two terms of the distribution

Sol: Given np=6 $\rightarrow$ (1) Mean = np

npq=2 $\rightarrow$ (2) variance = npq

$\frac{(2)}{(1)} \Rightarrow q=\frac{1}{3} \quad \therefore p=\frac{2}{3}$

From (1) $n\left(\frac{2}{3}\right)=6 \Rightarrow n=9$ $p(X=x)={}^{n}c_x p^x q^{n-x}$ (1M)

From two terms are $P(X=0)={}^9c_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^9=\frac{1}{3^9}$

$$P(X=1)={}^9c_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^8=\frac{9\times 2}{3^9}$$

$$=\boxed{\frac{2}{3^7}}$$ (1M)

7) **It is given that 10% of electric bulbs manufactured by a company are defective. In a sample of 20 bulbs find the probability that more than 2 are defective**

Sol: p=probability of defective bulb= $\frac{1}{10}$

$q=\frac{9}{10}$, n=20

Probability that more than 2 are defective (1M)

$$P(X>2)=\sum_{k=3}^{20}P(X=k)$$

$$=\sum_{k=3}^{20}{}^nc_kp^kq^{n-k}=\sum_{k=3}^{20}{}^{20}c_k\left(\frac{1}{10}\right)^k\left(\frac{9}{10}\right)^{20-k}$$

$$=\boxed{\sum_{k=3}^{20}{}^{20}c_k\frac{9^{20-k}}{10^{20}}}$$ **(1M)**

8) **Find constant C given F(*x*) =** $C.\left(\frac{2}{3}\right)^x$, x=1,2,3,............. **is p.d.f of a discrete random variable *x*.**

Sol: $\sum_{x=1}^{\infty}F(x)=1\Rightarrow\sum_{x=1}^{\infty}c.\left(\frac{2}{3}\right)^x=1$

$$\Rightarrow c\left[\frac{2}{3}+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+......\right]=1$$ **(1M)**

$$\Rightarrow c\left[\frac{\frac{2}{3}}{1-\frac{2}{3}}\right]=1 \qquad \therefore c\left[\frac{\frac{2}{3}}{\frac{1}{3}}\right]=1$$

$$\therefore \boxed{c=\frac{1}{2}}$$ (1M)

> $a+ar+ar^2$...........∞ is G.P.
>
> $s_\infty=\frac{a}{1-r}$
>
> Here $a=\frac{2}{3}$, $r=\frac{1}{3}$

Long answer questions **(7 Marks)**

Level-1 :

1) **The range of a random variable X is {0,1,2} given that P(X=0)=3c^3, P(X=1)=4c-10c^2, P(X=2)=5c-1, find (i) value of c (ii) P(X<1),** $P(1<X\le 2)$ **and** $P(0<X\le 3)$

Sol: We know that sum of probabilities=1

P(X=0)+P(X=1)+P(X=2)=1 **(1M)**

$3c^3+4c-10c^2+5c-1=1$

$3c^3-10c^2+9c-2=0$ By trial and error c=1

By synthetic division

1	3	-10	9	-2
	0	3	-7	-2
	3	-7	2	0

solving $3c^2-7c+2=0$ **(1M)**

$3c(c-2)-1(c-2)=0$

$(3c-1)(c-2)=0$

$\therefore C=2,\ c=\frac{1}{3}$

$\therefore$ c=1,2 are not possible ($\because P(X=0)>1$)

$\therefore\ c=\frac{1}{3}$ **(1M)**

(II) $P(X>1)=P(X=0)=3c^3=3\left(\frac{1}{3}\right)^3=\boxed{\frac{1}{9}}$ **(1M)**

$P(1<X\leq 2) = P(X=2)=5c-1= \frac{5}{3} -1 = \boxed{\frac{2}{3}}$ **(1M)**

$P(0<X\leq 3) = P(X=1)+P(X=2)$

$=4c-10c^2+5c-1$

$=9c-10c^2-1$

$= 9\left(\frac{1}{3}\right)-10\left(\frac{1}{9}\right)-1$

$= 3-\frac{10}{9}-1$ **(2M)**

$= \frac{8}{9}$

2) **A random variable X has the following probability distribution**

X=x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
P(X=x)	**0**	**k**	**2k**	**2k**	**3k**	**k^2**	**$2k^2$**	**$7k^2+k$**

Find (i) k value (ii) mean of x and (iii) P(0<X<5)

Sol: We know that sum of probabilities =1

$\sum_{i=0}^{7} P\left(x=x_i\right)=1$ **(1M)**

$\Rightarrow 0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$

$\Rightarrow 10k^2+9k-1=0$

$\Rightarrow 10k^2+10k-k-1=0$

$\Rightarrow 10k(k+1)-(k+1)=0$

$\Rightarrow (10k-1)(k+1)=0$

$\therefore \boxed{k=\frac{1}{10}}$ $(\because k\neq -1)$ **(1M)**

ii) Mean $\mu = \sum_{i=0}^{7} x_i p(x=x_i)$

$\mu=0(0)+1(k)+2(2k)+3(2k)+4(3k)+5(k^2)+6(2k^2)+7(7k^2+k)$

$66k^2+30k$

$\mu=66\left(\frac{1}{100}\right)+30\left(\frac{1}{10}\right)=\frac{366}{100}=3.66$ **(3M)**

iii) P(0<X<5)=P(X=1)+P(X=2)+P(X=3)+P(X=4)

=k+2k+2k+3k

=8k

$= \frac{8}{10} = \boxed{\frac{4}{5}}$ **(2M)**

3) A random variable X has the following probability distributions. Find k, mean and variance of X.

$X=x_i$	-2	-1	0	1	2	3
p(X=xi)	**0.1**	**k**	**0.2**	**2k**	**0.3**	**k**

Sol: We know sum of probabilities = 1

$\sum_{i=-2}^{3} p(X=x_i)=1$ **(1M)**

$\Rightarrow 0.1+k+0.2+2k+0.3+k=1$

$\Rightarrow 4k+0.6=1$

$\Rightarrow 4k=0.4$

$\Rightarrow \boxed{K=0.1}$ **(1M)**

Mean $\boxed{\mu = \sum_{i=-2}^{3} x_i p(x=x_i)}$

= -2(0.1)-1(k)+0(0.2)+1(2k)+2(0.3)+3(k)

= -0.2-k+2k+0.6+3k

= 4(0.1)+0.4

$\mu = \boxed{0.8}$ **(2M)**

Variance $\boxed{\sigma^2 = \sum_{i=-2}^{3} x_i^2 p(X=x_i)-\mu^2}$

$= 4(0.1)+1(k)+0(0.2)+1(2k)+4(0.3)+9(k)-(0.8)^2$

$= 0.4+k+2k+1.2k+9k-0.64$

$= 1.6+12(0.1)-0.64$

$= 1.6+1.2-0.64$ **(3M)**

$\sigma^2 = \boxed{2.16}$

4) A random variable X has the following probability distribution

$X=x_i$	1	2	3	4	5
$P(X=x_i)$	**k**	**2k**	**3k**	**4k**	**5k**

Find (i) k (ii) mean (iii) variance of *x*

Sol: sum of probability = 1

K+2k+3k+4k+5k=1

15k=1

$$k=\frac{1}{15}$$ **(2M)**

Mean $\mu=\sum_{i=1}^{5}x_i p(X=x_i)$

$= 1(k)+2(2k)+3(3k)+4(4k)+5(5k)$

= 55k

$= 55\left(\frac{1}{15}\right)=\frac{11}{3}$ **(2M)**

(III) variance $\sigma^2=\sum_{i=1}^{5}x_i^2 p(X=x_i)-\mu^2$

$= 1(k)+4(2k)+9(3k)+16(4k)+25(5k)-\frac{121}{9}$

$= 225k-\frac{121}{9}$

$=225\left(\frac{1}{15}\right)-\frac{121}{9}=15-\frac{121}{9}$

$=\frac{135-121}{9}=\frac{14}{9}$ **(3M)**

4) If X is a random variable with probability distribution $p(X=k)=\frac{(k+1)c}{2^k},k=0,1,2,.....$ **find c**

Sol: Given $p(X=k)=\frac{(k+1)c}{2^k},k=0,1,2,......$

$\sum_{k=0}^{\infty}p(X=k)=1$

$\Rightarrow \sum_{k=0}^{\infty}\frac{(k+1)c}{2^k}=1$ **(1M)**

$\Rightarrow \frac{c}{2^0}+\frac{2.c}{2^1}+\frac{3c}{2^2}+......=1$

$\Rightarrow c\left[1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2^2}\right)+....\right]=1$ **(2M)**

$\Rightarrow c\left(1-\frac{1}{2}\right)^{-2}=1$ $\left[\because (1-x)^{-2}=1+2x+3x^2+.....\infty\right]$

$\Rightarrow c\left(\frac{1}{2}\right)^{-2}=1$

$2^2.c=1 \Rightarrow c=\frac{1}{4}$ **(4M)**

Level-2

6) Two dice are rolled. Find the probability distribution of sum of numbers on them. Find mean of random variable.

Sol: When two dice are rolled the sample space s consists of $6\times6=36$ sample points

$S=\{(1,1)(1,2).....(1,6)(2,1)....(2,6)....(6,6)\}$

$\therefore n(S)=36$ **(1M)**

Let X denote sum of numbers on two dice

$\therefore$ range of X={2,3,4,5,6,7,8,9,10,11,12}

$X=x_i$	2	3	4	5	6	7	8	9	10	11	12
$P(X=x_i)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(2M)

Mean of $X=\mu=\sum_{i=2}^{12} x_i p(X=x_i)$

$=2\left(\frac{1}{36}\right)+3\left(\frac{2}{36}\right)+4\left(\frac{3}{36}\right)+5\left(\frac{4}{36}\right)+6\left(\frac{5}{36}\right)+7\left(\frac{6}{36}\right)+8\left(\frac{5}{36}\right)+9\left(\frac{4}{36}\right)+10\left(\frac{3}{36}\right)+11\left(\frac{2}{36}\right)+12\left(\frac{1}{36}\right)$

$=\frac{252}{36}=7$ □

7) One in 9 ships is likely to be wrecked when they are set on sail, when 6 ships are on sail. Find the probability of (i) atleast one will arrive safely (ii) exactly three will arrive safely

Sol: Let p, q be probabilities that ship arrive safely and likely to be wrecked respectively

$\therefore p=\frac{8}{9}$ $q=\frac{1}{9}$ n=6 **(2M)**

(i) Probability that atleast one ship will arrive safely

= 1-(prob. that no ship will arrive safely)

= 1-P(x=0)

$= 1-{}^6c_0.p^0.q^6$ $p(x\text{-}r)={}^nc_r p^r q^{n-r}$

$=1-\left(\frac{8}{9}\right)^0.\left(\frac{1}{9}\right)^6$

$=\boxed{1-\frac{1}{9^6}}$ **(3M)**

(ii) Probability that exactly 3 ships will arrive safely

$\Rightarrow P(x=3)={}^6c_3\left(\frac{8}{9}\right)^3\left(\frac{1}{9}\right)^3$

$=\boxed{20.\frac{8^3}{9^6}}$ **(2M)**

8) If the difference between mean and variance of a binomial variate is $\frac{5}{9}$ then find the probability for the event of 2 successes when the experiment is conducted 5 times

Sol: In binomial variate

Mean = np
Variance=npq

Given $np\text{-}npq = \frac{5}{9}$ and n=5 **(2M)**

$np(1\text{-}q)=\frac{5}{9}$ ($\because$ 1-q=p)

$np^2=\frac{5}{9} \Rightarrow p^2=\frac{1}{9} \Rightarrow p=\frac{1}{3}$

$\therefore q=\frac{2}{3}$ **(2M)**

Probability of two successes is P(X=2)

$={}^nc_2p^2q^{n-2}$

$={}^5c_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3$

$=10.\frac{1}{9}.\frac{8}{27}=\boxed{\frac{80}{243}}$ **(3M)**

9) In an experiment of tossing a coin n times, if variable x denotes number of heads and p(x=4),p(x=5) and p(x=6) are in A.P. Find n

Sol: Given $p=\frac{1}{2},q=\frac{1}{2}$

a,b,c are in A.P. $\Rightarrow$2b=a+c

Also p(x=4),q(x=5),p(x=6) are in A.P.

$\therefore 2p(x=5)=p(x=4)+p(x=6)$

$\Rightarrow 2,{}^nc_5\left(\frac{1}{2}\right)^5.\left(\frac{1}{2}\right)^{n-5}={}^nc_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{n-4}+{}^nc_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{n-6}$

$\boxed{p(x=r)={}^nc_rp^rq^{n-r}}$

$\Rightarrow 2{}^nc_5\frac{1}{2^n}={}^nc_4.\frac{1}{2^n}+{}^nc_6\frac{1}{2^n}$ **(2M)**

$\boxed{\frac{{}^nc_r}{{}^nc_{r-1}}=\frac{n-r+1}{r}}$

$\Rightarrow 2=\frac{{}^nc_4}{{}^nc_5}+\frac{{}^nc_6}{{}^nc_5}$ **(2M)**

$\Rightarrow 2=\frac{5}{n-4}+\frac{n-5}{6}$

$\Rightarrow 2=\frac{30+(n-4)(n-5)}{6(n-4)}$

$\Rightarrow 12n-48=30+n^2-9n+20$

$\Rightarrow n^2-21n+98=0$

$\Rightarrow (n-7)(n-14)=0$

$\therefore$ n=7 or n=14 **(3M)**

10) A cubical die is thrown. Find mean and variance of X, giving the number on the face that shows up

Sol: Let S be sample space and x the random variable P(X) is given by table

$X=x_i$	1	2	3	4	5	6	
$P(X=x_i)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	**(2M)**

mean of X $= \mu = \sum_{i=1}^{6} x_i p(\mathrm{x} = x_i)$

$= 1\left(\frac{1}{6}\right)+2\left(\frac{1}{6}\right)+3\left(\frac{1}{6}\right)+4\left(\frac{1}{6}\right)+5\left(\frac{1}{6}\right)+6\left(\frac{1}{6}\right)$

$= \frac{21}{6} = \boxed{\frac{7}{2}}$ **(2M)**

variance of $\mathrm{x} = \sigma^2 = \sum_{i=1}^{6} \mathrm{x_i^2 p(x=xi)} - \mu^2$

$\sigma^2 = 1\left(\frac{1}{6}\right)+4\left(\frac{1}{6}\right)+9\left(\frac{1}{6}\right)+16\left(\frac{1}{6}\right)+25\left(\frac{1}{6}\right)+36\left(\frac{1}{6}\right)-\frac{49}{4}$

$= \boxed{\frac{91}{6}} - \frac{49}{4} = \frac{35}{12}$ **(3M)**

Model Paper-I

Mathematics - IIA

Time: 3 Hours] **[Max. Marks: 75**

***Note*: The question paper consists of three sections A, B & C.**

Section-A

(10x2=20)

I. Very short answer question

Answer ALL questions. Each question carries two marks.

1) Find the multiplicative inverse of 7 + 24i
2) If $\text{Arg}\,\bar{Z}_1$ and $\text{Arg}Z_1$ are π and $\frac{\pi}{3}$ respectively then find $(\text{Arg}Z_1 + \text{Arg}Z_2)$
3) If I, W, W^2 are cube roots of unity then find the value of $(I\text{-}w+w^2)^5 + (I + w + w^2)^5$
4) Find the maximum or minimum of the expression $12x - x^2 - 32$.
5) Find the equation whose roots are reciprocals of the roots of $x^4 - 3x^3 + 7x^2 + 5x - 2 = 0$
6) If ${}^3P_3 = 1320$, find n.
7) Find the number of ways of arranging the letters of the word TRAINGLE so that relative positions of vowels and consonants are not disturbed.
8) Find the set of values of x of which $(7+3x)^{-5}$ is valid.
9) Find the mean deviation about median for the following data, 13, 17, 16, 11, 13, 10, 16, 11, 18, 12, 17.
10) A Poisson variable satisfies P(X=1) = P(X=2). Find the P(X=5).

Section – B

(5x4=20)

II. Short answer type questions:

i) Answer any **Five** questions.

ii) Each question carries **Four** marks.

11) If $x + iy = \dfrac{1}{1+\text{Cos}\theta + i\text{Sin}\,\theta}$ then show that $4x^2 - 1 = 0$.

12) If x is real then prove that $\dfrac{x}{x^2 - 5x + 9}$ lies between $-\dfrac{1}{11}$ and 1.

13) Find the sum of all 4 digit numbers that can be formed using digits 1, 3, 5, 7, 9 without reputation.

14) Find the number of ways of forming a committee of 5 members out of 6 Indians and 5 Americans so that Indians will be in majority in committee.

15) Resolve $\frac{x^3}{(x-a)(x-b)(x-c)}$ into partial fractions.

16) A, B, C are three horses in a race the probability of A to win the race is twice that of B and probability of B is twice that of C. What is probability of A, B and c to win the race.

17) A speaks truth is 75% of cases and B in 80% cases. What is the probability that their statements about an incident do not match.

Section – C
(5x7=35)

III. Long Answer Questions:

i) Answer any **Five** questions:

ii) Each question carries **Seven** marks.

18) If $Cos\alpha + Cos\beta + Cos\gamma = 0 = Sin\alpha + Sin\beta + Sin\gamma$. Then prove that

$$Cos^2\alpha + Cos^2\beta + Cos^2\gamma = \frac{3}{2} = Sin^2\alpha + Sin^2\beta + Sin^2\gamma$$

19) Solve $4x^3 - 24x^2 + 23x + 18 = 0$ given the roots of this equation are in A.P.

20) If 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(a+x)^n$ are respectively 240, 720, 1080, find a, x, n.

21) If $x = \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + ...\infty$ then find the value of $3x^2 + 6x$

22) Find the mean deviation from mean of the following data, using step deviation method.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	6	5	8	15	7	6	3

23) State and prove addition theorem on probability.

24) A random variable X has the following probability distribution.

X = x	0	1	2	3	4	5	6	7
P(X-x)	0	K	2k	2k	3k	K^2	$2k^2$	$7k^2$+k

Find (i) value of K, (ii) mean of x and (iii) P(0 < x < 5)

* * *

Model Paper-2
Mathematics - IIA

Time: 3 Hours] **[Max. Marks: 75**

***Note*: The question paper consists of three sections A, B & C.**

Section-A

(10x2=20)

III. Very short answer question

Answer ALL questions. Each question carries two marks.

25) Find the square root of 7 + 24i

26) If amplitude of (z-1) is $\frac{\pi}{2}$ then find locus of z.

27) If x = Cis θ find the value of $x^6 + \frac{1}{x^6}$

28) For what values of x the equation $x^2 + (m+3)x + (m+6) = 0$

29) If -1, 2, α are roots $2x^3 + x^2 - 7x - 6 = 0$ then find α

30) If ${}^nC_5 = {}^nC_6$ then find value of ${}^{13}C_n$

31) Find the number of ways of arranging 7 persons around a circle.

32) Find the number of terms in the expansion of $(2x+3y+z)^7$

33) Find the mean deviation from mean of the following discrete data 6, 7, 10, 12, 13, 4, 12, 16

34) The mean and variance of a binomial distribution are 6 and 2. Find the first two terms of the distribution.

Section – B

(5x4=20)

IV. Short answer type questions:

iii) Answer any **Five** questions.

iv) Each question carries **Four** marks.

35) Determine the locus of Z, $Z \neq 2i$ such that $\text{Re}\left(\frac{Z-4}{Z-2i}\right) = 0$

36) If the expression $\frac{x-p}{x^2-3x+2}$ takes all real values for $x \in \Re$ then find the bounds for P.

37) If the letters of the word EAMCET are permuted in all possible ways and if the words thus formed are arranged in dictionary order, find the rank of word EAMCET.

38) Find the number of ways of arranging 6 boys and 6 girls in a row. In how many of these arrangements

i) all girls are together

ii) boys and girls come alternately

39) Resolve $\frac{2x^2+3x+4}{(x-1)(x^2+2)}$ into partial fractions.

40) Let A and B be two independent events with P(A) = 0.2 P(B) = 0.5.

Find (a) P(A/B) (b) P(B/A) c) P(A $\cap$ B) d) P(A $\cup$ B)

41) Find the probability of drawing an arc of a space from a well shuffled pack of 52 playing cards,

Section – C

(5x7=35)

III. Long Answer Questions:

iii) Answer any **Five** questions:
iv) Each question carries **Seven** marks.

42) If n is an integer then show that $(1+i)^{2n}+(1-i)^{2n}=2^{n+1}\,Cos\left(\frac{n\pi}{2}\right)$

43) Solve $6x^4-35x^3+62x^2-35x+6=0$

44) If the coefficient of x^{10} in expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$ is equal to coefficient of x^{-10} is expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$ find the relation between a and b, then a, b are real numbers.

45) Find the sum of infinite series $\frac{3}{4}+\frac{3.5}{4.8}+\frac{3..7}{4.8.12}+.............\infty$

46) Calculate the variance and standard deviation for the given discrete frequency distribution.

xi	4	8	11	17	20	24	32
fi	3	5	9	5	4	3	1

47) State and prove Baye's theorem.

48) The range of a random variable x is {0, 1, 2} given that P(x=0) = 3c^3, P(x=1)=4c-10c^2, P(x=2)=5c-1

i) Find value of c ii) P(x < 1), P(1 < x $\leq$2) and P(0 < x $\leq$ 3).

* * *

TSWREIS, Hyderabad
Model Paper
Mathematics – IIA
Sr. MPC

Time: 3 Hours] **[Max. Marks: 75**

***Note*: The question paper consists of three sections A, B & C.**

Section-A

(10x2=20)

V. Solve the TEN (10) problems:

49) Find the complex conjugate of (3+4i) (2-3i)

50) If z_1 = -1, z_2 = I then find Avg $\left(\frac{z_1}{z_2}\right)$

51) Find the value of $(1+\bar{i})^{16}$

52) If α, β are the roots of the equation $ax^2 + bx + c = 0$. Then find the value of $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

53) If the product of $4x^3 + 16x^2 – 9x – a = 0$, is 9 then find a

54) Find the number of ways of arranges the letter of the word INDEPENDENCE.

55) ${}^{n}p_{r} = 42 \text{ x } {}^{n}p_{5}$ find n

56) Find the number of terms in the expansion of $(2x+3y+z)^7$.

57) Find the variance and standard deviation the following data 5, 12, 3, 18, 6, 8, 2, 10.

58) The probability that the person chosen at random is left handed is 0.1 what is the probability that in a group of 10 people. There in one who is left handed.

Section – B

(5x4=20)

VI. Solve 5 questions:

59) Show that the pts in the argand plane represented by the complex number -2+7i, $-\frac{3}{2}+\frac{1}{2}i$, 4-3i, $\frac{7}{2}$ (1+i) are vertices of a Rhombus.

60) If x is real, prove that $\frac{x}{x^2-5x+9}$ lines between $-\frac{1}{11}$, 1

61) If the letters of the word PRISON are permuted in all possible ways and the words thus formed are arranged in dictionary order, find the rank of the word PRISON.

62) Find the number of ways of selection a cricket team of 11 players from 7 batsman and 6 bowlers such of 11 players from 7 batsman and 6 bowlers such that there be at least 5 bowlers in the team.

63) Resolve $\frac{x^3}{(2x-1)(x-1)}$

64) State and prove the multiplication theorem of probability.

65) A & B are events with P(A) = 0.5, P(B) = 0.4 and $P(P \cap B) = 0.3$. Find the probability that i) A does not occur ii) neither A nor B occurs.

Section – C
(5x7=35)

III. Solve 5 questions:

18) Show that one value of $\left[\frac{1+\sin\frac{\pi}{8}+i\cos\frac{T}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right]$ is – 1

19) Find the equation whose revts translation the roots of $x^5+4x^3-x^2+16=0$ by -3.

20) If the constant of 4 consecution theorem in the proven of $(1+x)^n$ are a_1 a_2 a_3 a_4 respectively then show that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}+$

21) If $x=\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+$ ……. ….. then prove that $9x^2+24x=11$.

22) Find the mean deviation about the mean for the following continuous distribution

Height	95-105	105-115	115-125	125-135	135-145	145-155
No. of boys	9	13	26	30	12	10

23) State and prove Baye's theorem.

24) Random variable x has the following probability distribution.

x=xi	1	2	3	4	5
P=(x=xi)	K	2K	3K	4K	5K

Find i) K ii) mean and iii) variance of x.

* * *